METRO
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Mass transport at the solid/liquid interface of growing composite *in situ*

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Education and Culture

Microstructure of the growing composite *in situ*

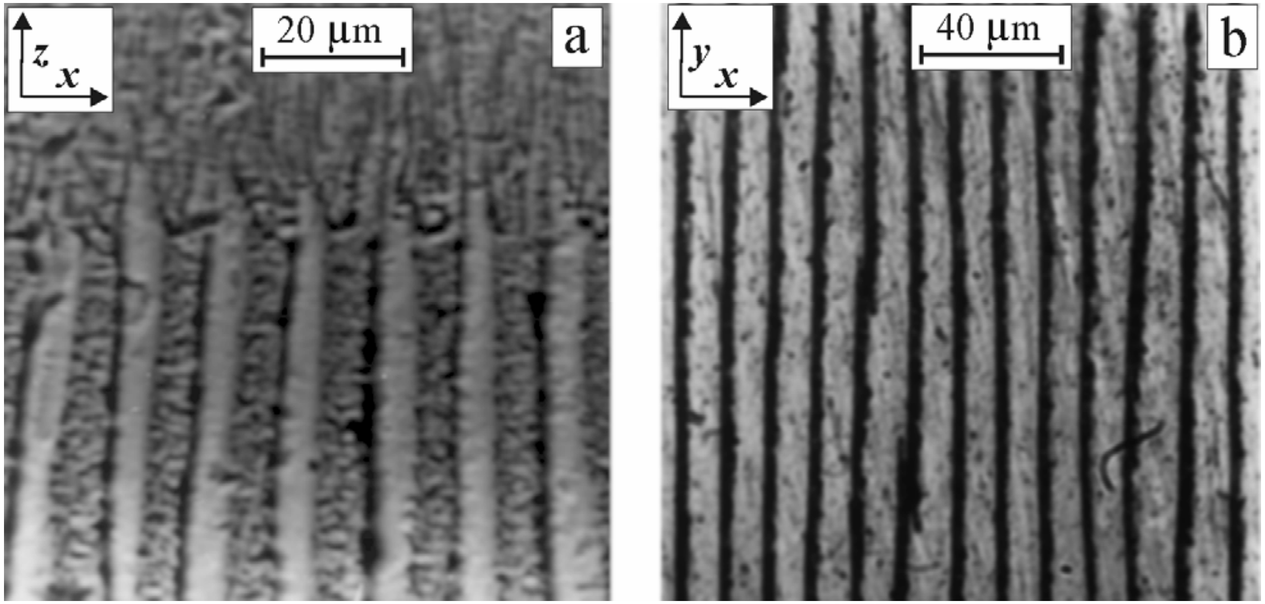


FIG. 1

Oriented growth of the (Pb) – (Cd) composite *in situ*
a/ frozen solid/liquid interface (s / l interface)
b/ typical structure of growing α / β composite *in situ*

Conditions applied to the diffusion equation by **Jackson-Hunt** theory

$$\nabla^2 C + \frac{v}{D}\frac{\partial C}{\partial z} = 0$$

$\frac{\partial C}{\partial x} = 0$ at $x = 0$ and $x = S_\alpha + S_\beta$

$\frac{\partial C}{\partial z} = -\frac{v C_0^\alpha}{D}$ for $0 \le x < S_\alpha$

$\frac{\partial C}{\partial z} = +\frac{v C_0^\beta}{D}$ for $S_\alpha < x \le S_\alpha + S_\beta$

C – solute concentration in the liquid at the solid/liquid interface

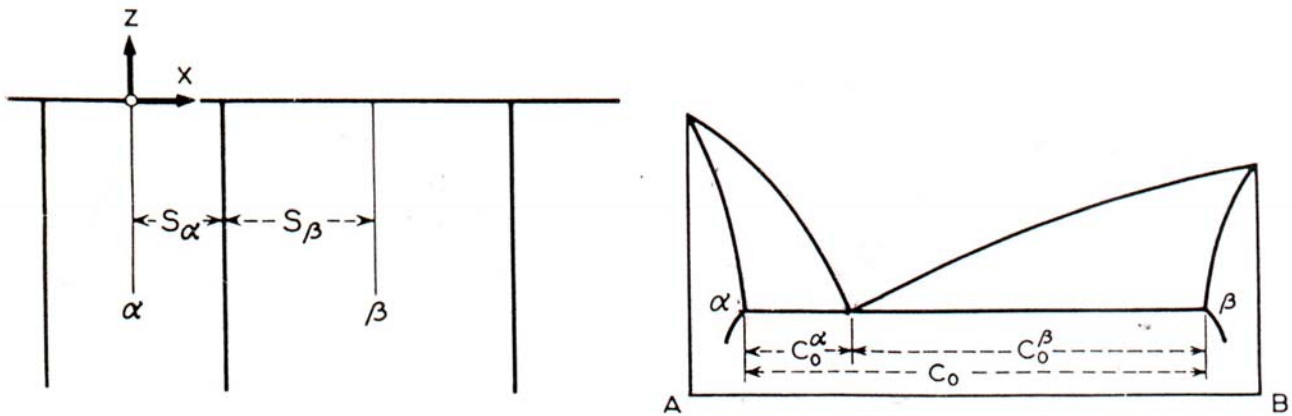
D – coefficient of diffusion in the liquid

K.A. Jackson, J.D. Hunt, Trans. AIME, **236**, 1129-1142, (1966)

Solution to the diffusion equation by **Jackson - Hunt** theory

FIG. 2

$$C = C_E + C_\infty + \sum_{n=1}^{\infty} B_n \cos\left(\frac{\pi n x}{S_\alpha + S_\beta}\right) \exp\left(-\frac{\pi n z}{S_\alpha + S_\beta}\right)$$



with

$$B_0 = \frac{C_0^\alpha S_\alpha - C_0^\beta S_\beta}{S_\alpha + S_\beta}$$

Composite *in situ* growth:
a/ J-H planar s/l interface
b/ adequate J-H phase diagram

K.A. Jackson, J.D. Hunt,
Trans. AIME, **236**, 1129-1142, (1966)

A solid / liquid interface behaviour according to Jackson-Hunt theory

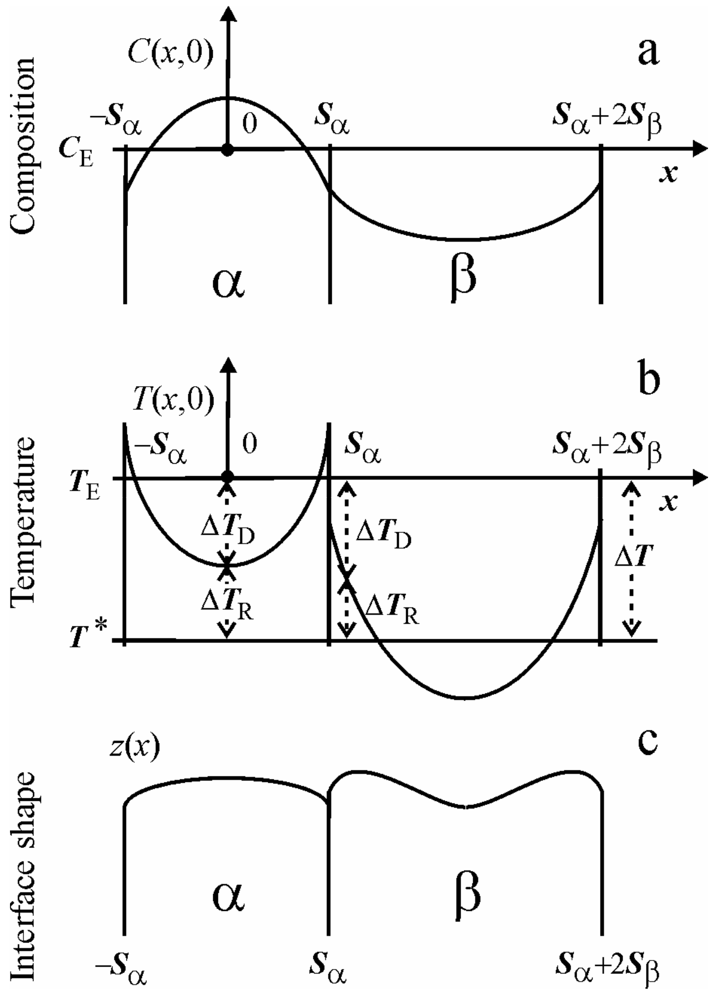


FIG.3

Imperfections of the J-H theory

a/ no mass balance:
$C(x,0) - C_E$ for α phase lamella is not equal to $C_E - C(x,0)$ for β phase lamella

b/ mass balance is satisfied for $S_\alpha = S_\beta$ and $C_0^\alpha = C_0^\beta$, only, in J-H theory

c/ undercooling greater than $\Delta T = T^* - T_E$ assumed in concept of *ideally coupled growth*

d/ discontinuity of temperature at the α / β inter-phase boundary

e/ non-realistic curvature of the solid / liquid interface shape

K.A. Jackson, J.D. Hunt, Trans. AIME, **236**, 1129-1142, (1966)

A solid / liquid interface behaviour according to Jackson-Hunt theory

according to the J-H scheme (FIG. 3a) some parts of α - phase should grow from the liquid of the solute concentration adequate to the formation of β - phase, rather it could lead to the changes in inter-lamellar spacing and instability of a solid/liquid interface

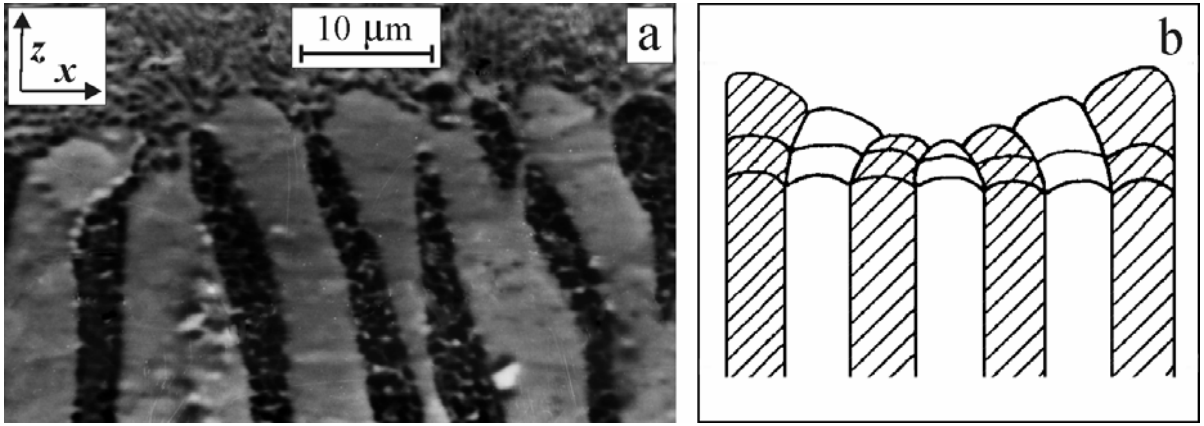
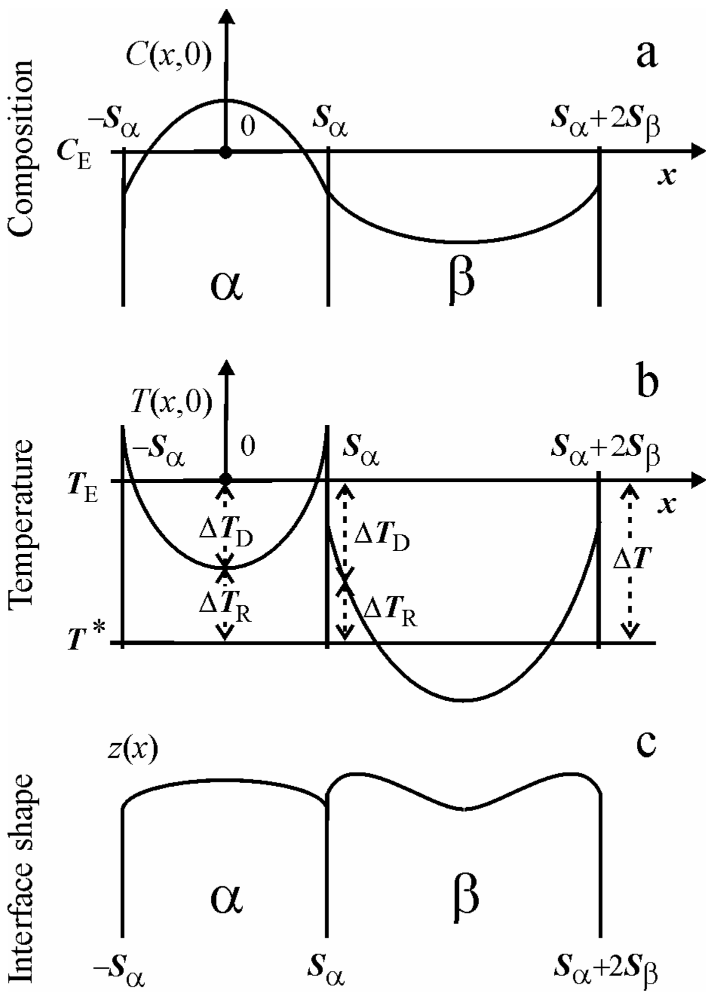


FIG. 4

instability at the solid / liquid interface
a/ observed during composite *in situ* growth
b/ concluded from the J-H theory (as above)

K.A. Jackson, J.D. Hunt, Trans. AIME, **236**, 1129-1142, (1966)

A solid / liquid interface behaviour according to Jackson-Hunt theory



according to J-H solution of diffusion equation concentration profile is common for both lamellae of the composite *in situ*

^assumption of non separation of concentration micro-field (in the J-H theory)
together with concept of *ideally coupled growth*
$\Delta T_\alpha^* = \Delta T_\beta^* = \Delta T$ results in discontinuity of undercooling at α / β inter-phase boundary
^moreover, some parts of the α as well as β phase lamellae should grow outside of the regime

FIG.5

undercooling
ΔT_D results from changes of the solute concentration
ΔT_R results from the s / l interface curvature

Application of Jackson-Hunt theory to the phase diagram

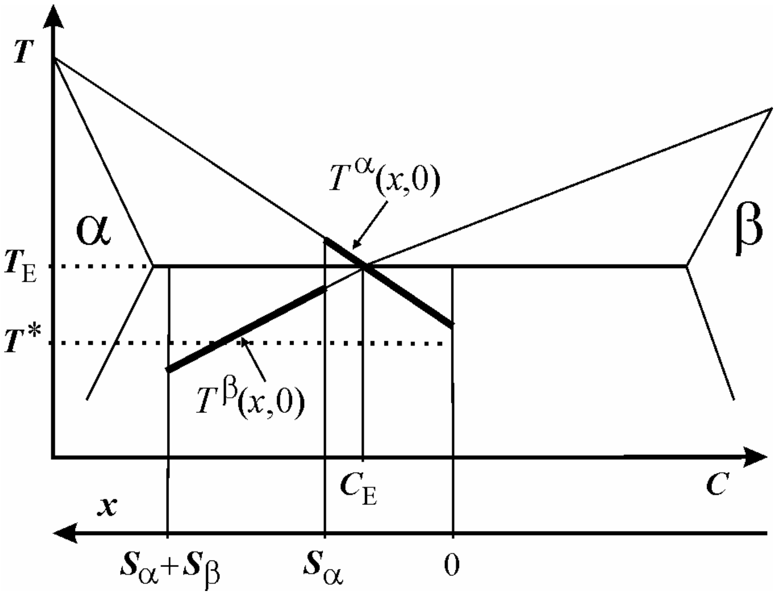


FIG.6

total undercooling: $\Delta T = T^* - T_E$

C	**solute concentration**
C_E	**solute concentration at eutectic point**
S_α	**half the width of the α phase lamella**
S_β	**half the width of the β phase lamella**
T	**equilibrium temperature**
$T^\alpha(x,z)$	**equilibrium temperature corresponding to changes in solute concentration at the α phase solid / liquid interface (z = 0)**
$T^\beta(x,z)$	**equilibrium temperature corresponding to changes in solute concentration at the β phase solid / liquid interface (z = 0)**
T^*	**real temperature of the solid/liquid interface**
T_E	**temperature of eutectic transformation**

undercooling – phase diagram

K.A. Jackson, J.D. Hunt, Trans. AIME, **236**, 1129-1142, (1966)

An initial correction of Jackson-Hunt theory

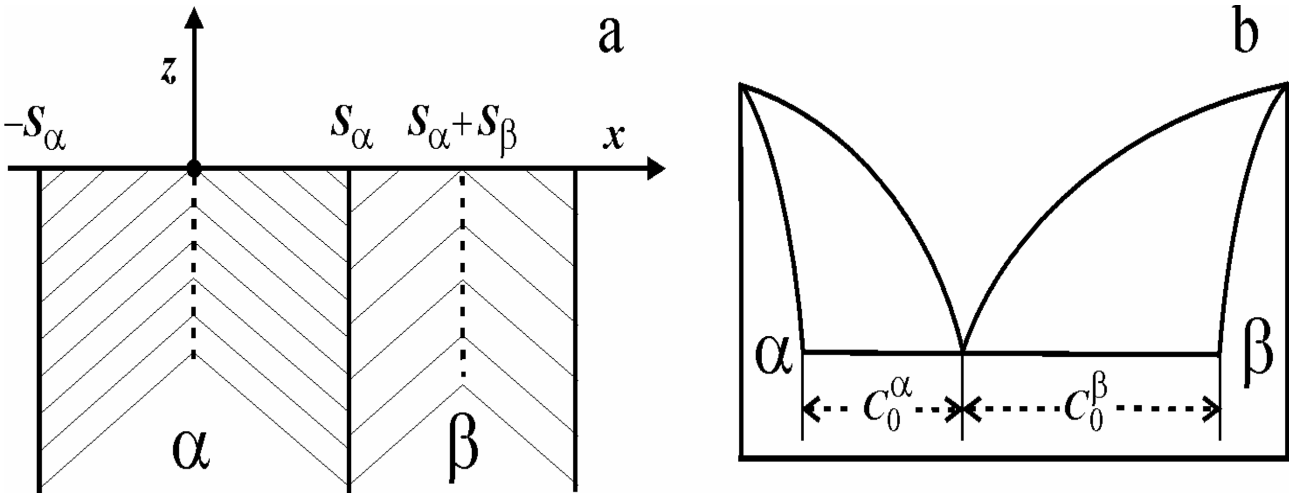


FIG. 7

now

$$C_0^\alpha S_\alpha = C_0^\beta S_\beta$$

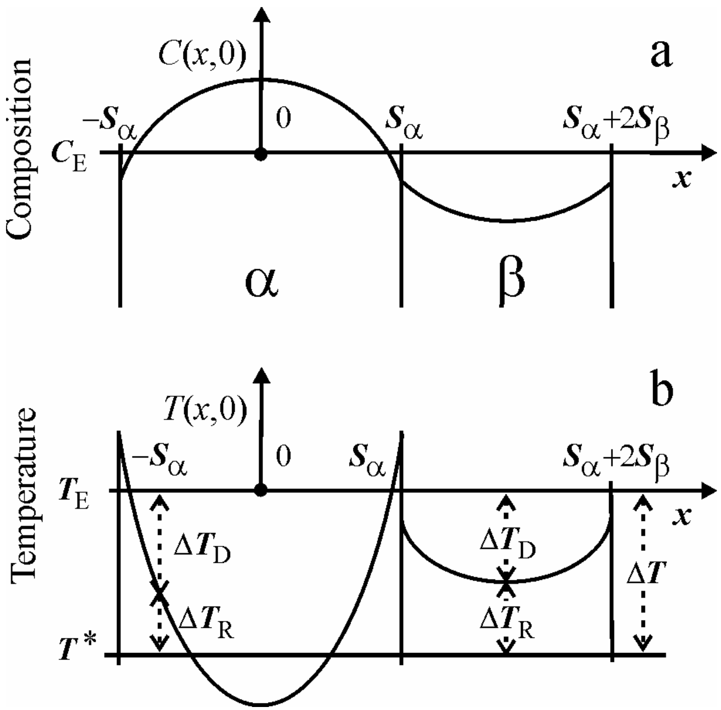
thus

a/corrected planar solid / liquid interface
b/corresponding arbitrary phase diagram

$$B_0 = \frac{C_0^\alpha S_\alpha - C_0^\beta S_\beta}{S_\alpha + S_\beta} = 0$$

$$C = C_E + C_\infty + \sum_{n=1}^{\infty} B_n \cos\left(\frac{\pi n x}{S_\alpha + S_\beta}\right) \exp\left(-\frac{\pi n z}{S_\alpha + S_\beta}\right)$$

Initially corrected Jackson - Hunt theory
Solute concentration and undercooling



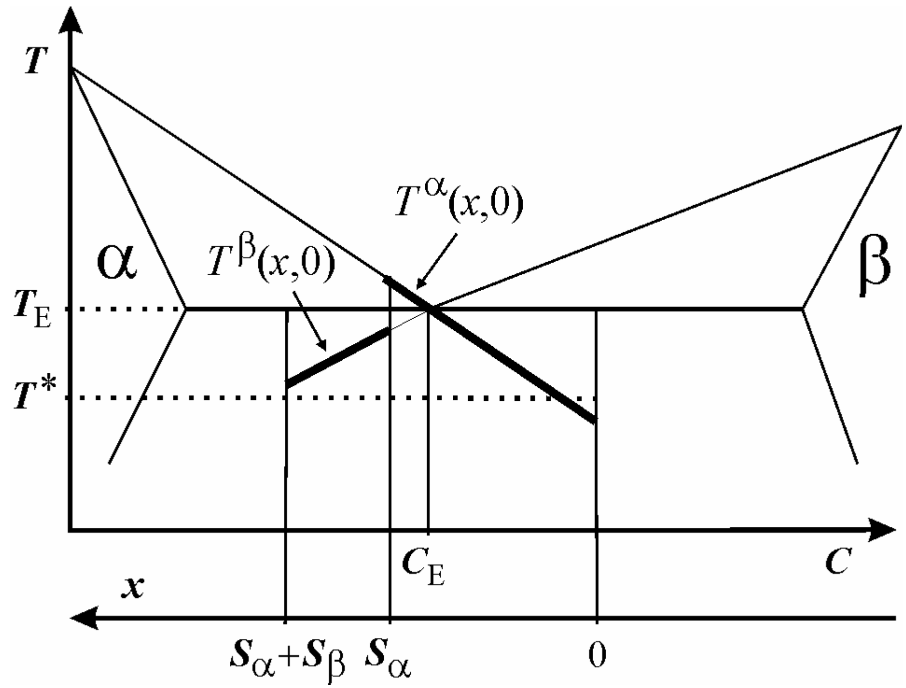
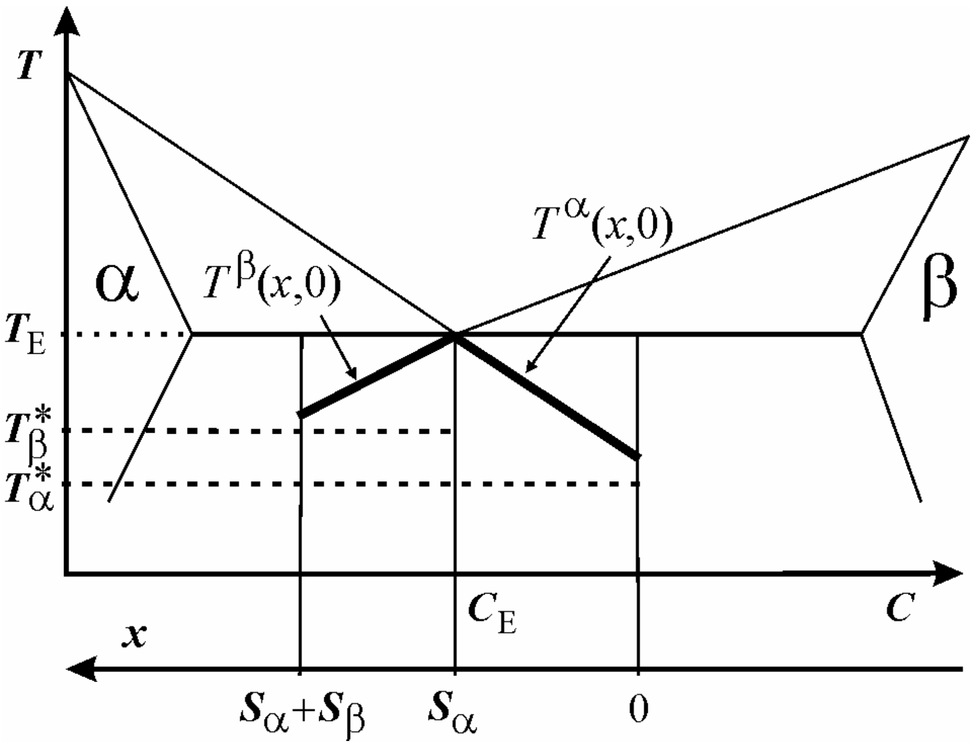


FIG. 8 solute concentration and undercooling

FIG. 9 undercooling – phase diagram

no significant improvement of the J-H theory ! - as it is seen in FIG. 9 (to be compared with the scheme in FIG. 6)

Concept of the *coupled growth*



***coupled growth* is a new concept to improve J-H theory**

$$\Delta T^{*}_{\alpha} = T^{*}_{\alpha} - T_{E}$$

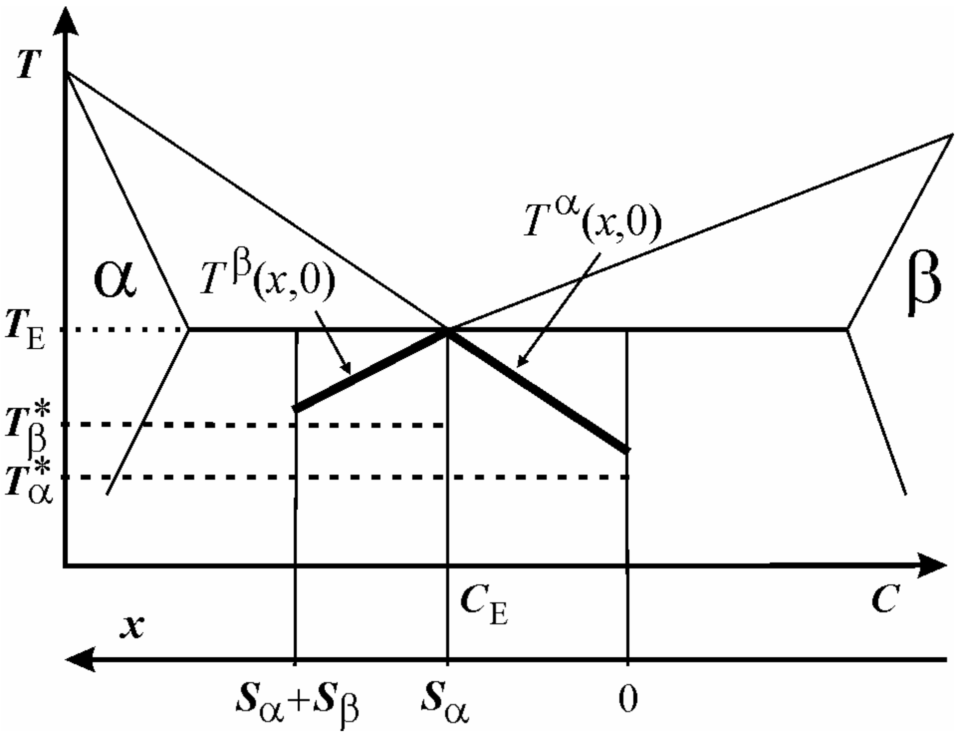
$$\Delta T^{*}_{\beta} = T^{*}_{\beta} - T_{E}$$

desired relation: undercooling – phase diagram

FIG. 10

but generally ➔ $\Delta T_{\alpha}^{*} \neq \Delta T_{\beta}^{*}$

Undercooling of the s/l interface due to the concept of *coupled growth*



curvature undercooling

$$\delta T_R^{\alpha}(x,0) = T_\alpha^* - T^{\alpha}(x,0)$$

$$\delta T_R^{\beta}(x,0) = T_\beta^* - T^{\beta}(x,0)$$

undercooling – phase diagram

total undercooling ➔

$$\delta T^{\alpha}(x,0) + \delta T_R^{\alpha}(x,0) = \Delta T_\alpha^*$$

$$\delta T^{\beta}(x,0) + \delta T_D^{\beta}(x,0) = \Delta T_\beta^*$$

FIG. 11

Solute concentration due to the concept of *coupled growth*

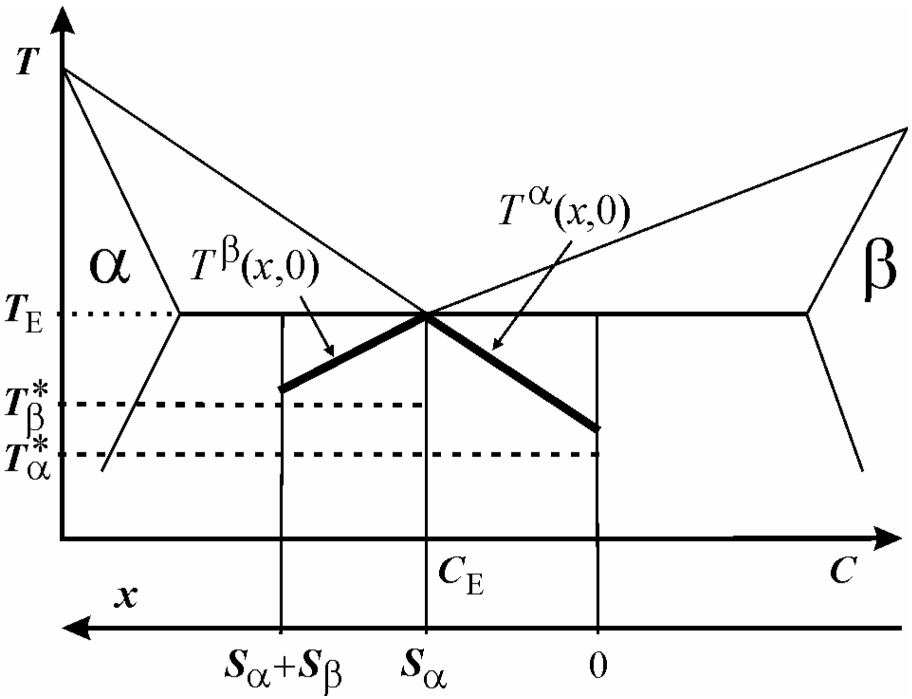


FIG. 12

$$\delta C^{\alpha}(x,0) = C^{\alpha}(x,0) - C_E$$

$$\delta C^{\beta}(x,0) = C^{\beta}(x,0) - C_E$$

undercooling – phase diagram

consequentially ➔

$$C_0^{\alpha}(S_{\alpha},0) = C_S^{\alpha}(S_{\alpha},0) - C_E < 0$$

$$C_0^{\beta}(S_{\alpha},0) = C_S^{\beta}(S_{\alpha},0) - C_E > 0$$

Fundamentals of the new solution due to the concept of *coupled growth*

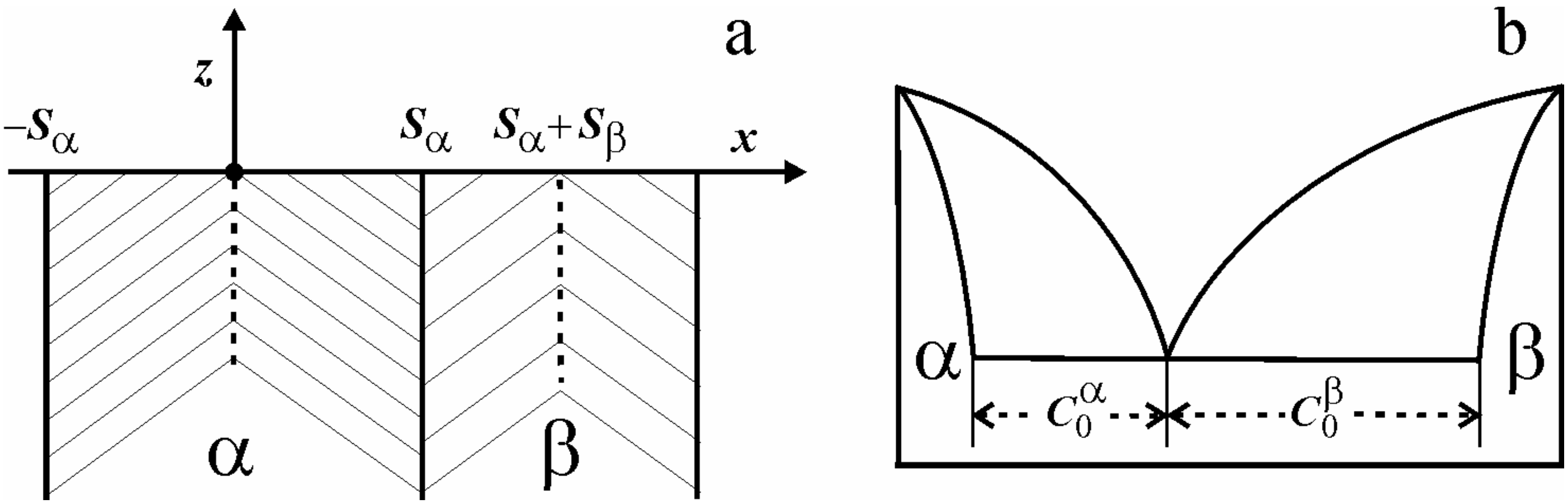


FIG. 13

a/ planar s / l interface
b/ corresponding
arbitrary phase diagram

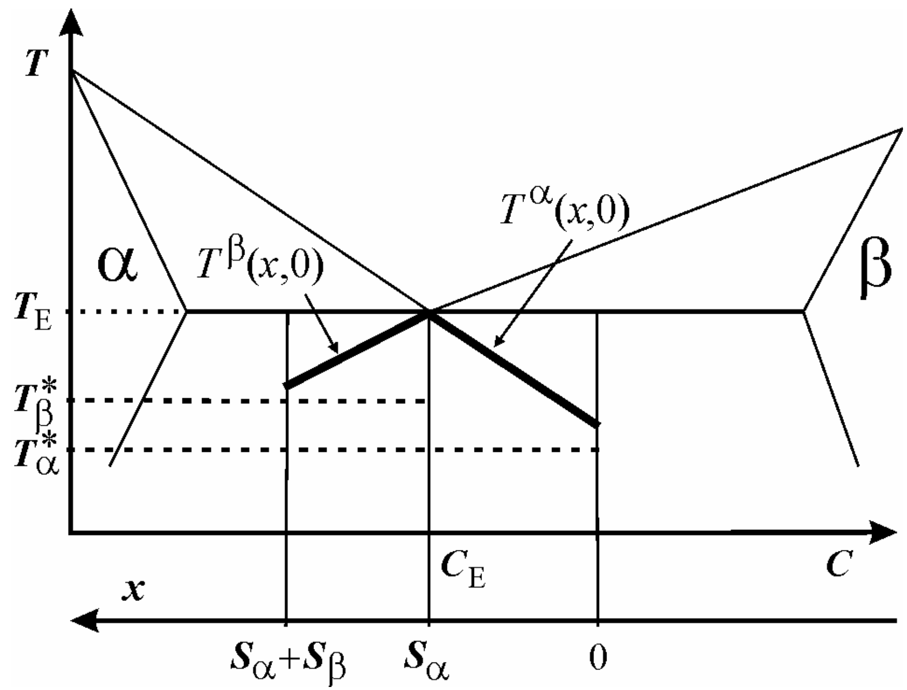
$$C_0^\alpha(S_\alpha,0)=C_S^\alpha(S_\alpha,0)-C_E<0$$

$$C_0^\beta(S_\alpha,0)=C_S^\beta(S_\alpha,0)-C_E>0$$

Diffusion equation due to the concept of *coupled growth*

FIG. 14

undercooling – phase diagram



diffusion equation is:

$$\frac{\partial^2 \delta C}{\partial x^2}+\frac{\partial^2 \delta C}{\partial z^2}+\frac{v}{D}\frac{\partial \delta C}{\partial z}=0$$

with

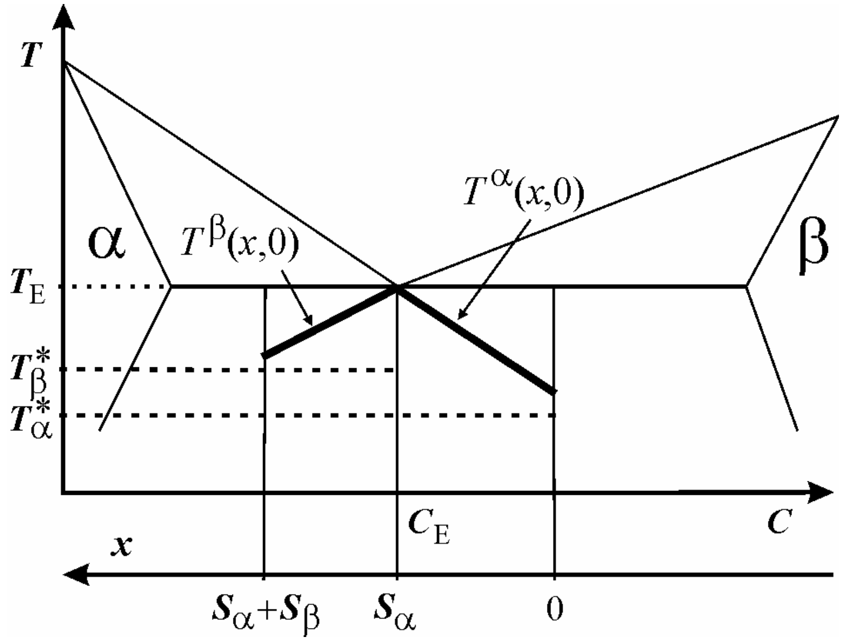
$$\delta C(S_\alpha, z)=C(S_\alpha, z)-C_E=0$$

Assumptions for the new solution to diffusion equation

undercooling – phase diagram

diffusion equation is:

$$\frac{\partial^2 \delta C}{\partial x^2} + \frac{\partial^2 \delta C}{\partial z^2} + \frac{v}{D}\frac{\partial \delta C}{\partial z} = 0$$



it is required to solve the diffusion equation in such a way to:
a/ satisfy the solid / liquid interface undercooling behaviour defined in FIG. 15
b/ obtain the solution separately for the α – phase lamella and β – phase lamella

FIG. 15

General solution to the diffusion equation

diffusion equation is:

$$\frac{\partial^2 \delta C}{\partial x^2} + \frac{\partial^2 \delta C}{\partial z^2} + \frac{v}{D}\frac{\partial \delta C}{\partial z} = 0$$

general solution to diffusion equation formulated in accordance with the concept of coupled growth is as follows:

$$\delta C(x,z) = X(x)\,Z(z)$$

Solution to the diffusion equation
Detailed formulations

general solution to diffusion equation formulated in accordance with the concept of coupled growth is as follows:

$$\delta C(x,z) = X(x)\, Z(z)$$

where

$$X(x) = A\cos(\omega x) + B\sin(\omega x)$$

$$Z(z) = \exp\left[\left(-\frac{v}{2D} - \sqrt{\frac{v^2}{4D^2} + \omega^2}\right) z\right]$$

A, B, ω - parameters are to be defined

Definition of the unknown parameters

A, B, ω - parameters are to be defined

definitions are to be given separately

a/ for the α phase lamella $x \in [0, S_\alpha]$ $z \geq 0$

the values of B and ω parameter yield from conditions a/ and b/:

a/ $\left.\frac{\partial \delta C(x,z)}{\partial x}\right|_{x=0} = 0$ and b/ $\delta C(S_\alpha, z) = 0$

from a/ $-\omega A \sin(\omega \cdot 0) + \omega B \cos(\omega \cdot 0) = 0$ it yields $B = 0$

from a/ and b/ $A \cos(\omega S_\alpha) = 0$

and $\omega = \omega_{2n-1} = \frac{(2n-1)\pi}{2S_\alpha}, \quad n = 1,2,\ldots$

Detailed solution to the diffusion equation

a/ for the α phase lamella $x \in [0, S_\alpha]$ $z \geq 0$

the detailed solution to diffusion equation is:

$$\delta C(x,z) = \sum_{n=1}^{\infty} A_{2n-1} \cos\left(\frac{(2n-1)\pi x}{2S_\alpha}\right) \exp\left[\left(-\frac{v}{2D} - \sqrt{\frac{v^2}{4D^2} + \left(\frac{(2n-1)\pi}{2S_\alpha}\right)^2}\right) z\right]$$

where A_{2n-1} are constants

Slow solidification
Solution to diffusion equation

it is evident that for slow solidification:

$$\frac{(2n-1)\pi}{2S_\alpha} >> \frac{v}{2D}$$

a/ for the α phase lamella $x \in [0, S_\alpha]$ $z \geq 0$

$$\delta C(x,z) = \sum_{n=1}^{\infty} A_{2n-1} \cos\left(\frac{(2n-1)\pi x}{2S_\alpha}\right) \exp\left[\left(-\frac{v}{2D} - \sqrt{\frac{v^2}{4D^2} + \left(\frac{(2n-1)\pi}{2S_\alpha}\right)^2}\right) z\right]$$

reduces to

$$\delta C(x,z) = \sum_{n=1}^{\infty} A_{2n-1} \cos\left(\frac{(2n-1)\pi x}{2S_\alpha}\right) \exp\left(-\frac{(2n-1)\pi}{2S_\alpha} z\right)$$

A_{2n-1} parameters

the values of A_{2n-1} parameters are calculated applying the condition:

$$\left.\frac{\partial \delta C(x,z)}{\partial z}\right|_{z=0} = f_\alpha(x); \quad f_\alpha(x) < 0, \quad x \in [0, S_\alpha]$$

for

a/ micro-filed of solute concentration for rapid solidification

$$\left.\frac{\partial \delta C(x,z)}{\partial z}\right|_{z=0} = \sum_{n=1}^{\infty} A_{2n-1}\left(-\frac{v}{2D} - \sqrt{\frac{v^2}{4D^2} + \left(\frac{(2n-1)\pi}{2S_\alpha}\right)^2}\right)\cos\left(\frac{(2n-1)\pi x}{2S_\alpha}\right)$$

b/ micro-filed of solute concentration for slow solidification

$$\left.\frac{\partial \delta C(x,z)}{\partial z}\right|_{z=0} = \sum_{n=1}^{\infty} A_{2n-1}\left(-\frac{(2n-1)\pi}{2S_\alpha}\right)\cos\left(\frac{(2n-1)\pi x}{2S_\alpha}\right)$$

Application of the f(x) function

additionally, new function f(x) is to be introduced:

$$f(x), \quad -2S_\alpha \le x \le 2S_\alpha, \quad f(-x) = f(x), \quad f(x+2S_\alpha) = -f(x)$$

the following property of the f(x) is to be applied

$$f(x) \approx \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{2S_\alpha}\right)$$

where

$$a_n = \frac{1}{S_\alpha} \int_0^{2S_\alpha} f(x) \cos\left(\frac{n\pi x}{2S_\alpha}\right) dx$$

f(x)

since: $f(x+2S_\alpha) = -f(x)$ for $n = 2k$ $k = 0, 1, 2, \dots$

it yields:

$$a_{2k} = \frac{1}{S_\alpha}\int_0^{2S_\alpha} f(x)\cos\left(\frac{2k\pi x}{2S_\alpha}\right)dx =$$

$$\frac{1}{S_\alpha}\left(\int_0^{S_\alpha} f(x)\cos\left(\frac{2k\pi x}{2S_\alpha}\right)dx + \int_{S_\alpha}^{2S_\alpha} f(x)\cos\left(\frac{2k\pi x}{2S_\alpha}\right)dx\right) =$$

$$\frac{1}{S_\alpha}\left(\int_0^{S_\alpha} f(x)\cos\left(\frac{2k\pi x}{2S_\alpha}\right)dx + \int_{-S_\alpha}^{0} f(x+2S_\alpha)\cos\left(\frac{2k\pi(x+2S_\alpha)}{2S_\alpha}\right)dx\right) =$$

$$\frac{1}{S_\alpha}\left(\int_0^{S_\alpha} f(x)\cos\left(\frac{2k\pi x}{2S_\alpha}\right)dx - \int_{-S_\alpha}^{0} f(x)\cos\left(\frac{2k\pi x}{2S_\alpha}\right)dx\right) =$$

$$\frac{1}{S_\alpha}\left(\int_0^{S_\alpha} f(x)\cos\left(\frac{2k\pi x}{2S_\alpha}\right)dx + \int_{0}^{-S_\alpha} f(x)\cos\left(\frac{2k\pi x}{2S_\alpha}\right)dx\right) =$$

$$\frac{1}{S_\alpha}\left(\int_0^{S_\alpha} f(x)\cos\left(\frac{2k\pi x}{2S_\alpha}\right)dx + \int_{0}^{S_\alpha} f(-x)\cos\left(\frac{-2k\pi x}{2S_\alpha}\right)d(-x)\right) =$$

$$\frac{1}{S_\alpha}\left(\int_0^{S_\alpha} f(x)\cos\left(\frac{2k\pi x}{2S_\alpha}\right)dx - \int_{0}^{S_\alpha} f(x)\cos\left(\frac{2k\pi x}{2S_\alpha}\right)dx\right) = 0$$

f(x)

since: $f(x+2S_\alpha) = -f(x)$ for $n = 2k-1,\ k = 1,2,\ldots$

it yields:

$$a_{2k-1} = \frac{1}{S_\alpha}\int_0^{2S_\alpha} f(x)\cos\left(\frac{(2k-1)\pi x}{2S_\alpha}\right)dx =$$

$$\frac{1}{S_\alpha}\left(\int_0^{S_\alpha} f(x)\cos\left(\frac{(2k-1)\pi x}{2S_\alpha}\right)dx + \int_{S_\alpha}^{2S_\alpha} f(x)\cos\left(\frac{(2k-1)\pi x}{2S_\alpha}\right)dx\right) =$$

$$\frac{1}{S_\alpha}\left(\int_0^{S_\alpha} f(x)\cos\left(\frac{(2k-1)\pi x}{2S_\alpha}\right)dx + \int_{-S_\alpha}^{0} f(x+2S_\alpha)\cos\left(\frac{(2k-1)\pi (x+2S_\alpha)}{2S_\alpha}\right)dx\right) =$$

$$\frac{1}{S_\alpha}\left(\int_0^{S_\alpha} f(x)\cos\left(\frac{(2k-1)\pi x}{2S_\alpha}\right)dx - \int_{-S_\alpha}^{0} f(x)\cos\left(\frac{(2k-1)\pi x}{2S_\alpha} + 2k\pi + \pi\right)dx\right) =$$

$$\frac{1}{S_\alpha}\left(\int_0^{S_\alpha} f(x)\cos\left(\frac{(2k-1)\pi x}{2S_\alpha}\right)dx + \int_{-S_\alpha}^{0} f(x)\cos\left(\frac{(2k-1)\pi x}{2S_\alpha}\right)dx\right) =$$

$$\frac{1}{S_\alpha}\left(\int_0^{S_\alpha} f(x)\cos\left(\frac{(2k-1)\pi x}{2S_\alpha}\right)dx - \int_{0}^{-S_\alpha} f(x)\cos\left(\frac{(2k-1)\pi x}{2S_\alpha}\right)dx\right) =$$

$$\frac{1}{S_\alpha}\left(\int_0^{S_\alpha} f(x)\cos\left(\frac{(2k-1)\pi x}{2S_\alpha}\right)dx - \int_{0}^{S_\alpha} f(-x)\cos\left(\frac{-(2k-1)\pi x}{2S_\alpha}\right)d(-x)\right) =$$

$$\frac{2}{S_\alpha}\int_0^{S_\alpha} f(x)\cos\left(\frac{(2k-1)\pi x}{2S_\alpha}\right)dx$$

Result of the application of the f(x) function

assuming: $f(x+2S_\alpha) = -f(x)$

it yields $a_{2k} = 0$ $k = 0, 1, 2, \ldots$ for $n = 2k$

and

$$a_{2k-1} = \frac{2}{S_\alpha} \int_0^{S_\alpha} f(x) \cos\left(\frac{(2k-1)\pi x}{2S_\alpha} \right) dx$$

for $n = 2k-1, k = 1, 2, \ldots$

finally, the *Fourier* series of the f(x) is:

$$f(x) \approx \sum_{k=1}^{\infty} a_{2k-1} \cos\left(\frac{(2k-1)\pi x}{2S_\alpha} \right)$$

Final definition of the A_{2n-1} parameter

for rapid solidification:

$$A_{2n-1} = \left(-\frac{v}{2D} - \sqrt{\frac{v^2}{4D^2} + \left(\frac{(2n-1)\pi}{2S_\alpha} \right)^2} \right)^{-1} \frac{2}{S_\alpha} \int_0^{S_\alpha} f_\alpha(x) \cos\left(\frac{(2n-1)\pi\, x}{2S_\alpha} \right) dx$$

$n = 1, 2, \ldots$

for slow solidification:

$$A_{2n-1} = -\frac{4}{(2n-1)\pi} \int_0^{S_\alpha} f_\alpha(x) \cos\left(\frac{(2n-1)\pi\, x}{2S_\alpha} \right) dx$$

$n = 1, 2, \ldots$

Properties of the solution to diffusion equation

$$\left.\frac{\partial \delta C(x,z)}{\partial x}\right|_{x=0} = \left.\frac{\partial \delta C(x,z)}{\partial x}\right|_{x=2S_\alpha} = 0 \qquad x \in [0, S_\alpha]$$

$$\left.\frac{\partial \delta C(x,z)}{\partial z}\right|_{z=0} = f_\alpha(x) = f_\alpha(-x) = -f_\alpha(-x+2S_\alpha) = -\left.\frac{\partial \delta C(-x+2S_\alpha, z)}{\partial z}\right|_{z=0}$$

according to assumption:

$$f_\alpha(-x) = f_\alpha(x), \qquad f_\alpha(x+2S_\alpha) = -f_\alpha(x)$$

Solution to the diffusion equation

b/ for the β phase lamella $x \in \left[S_\alpha, S_\alpha + S_\beta\right]$ $z \geq 0$

$$\delta C(x,z) =$$

$$\sum_{n=1}^{\infty} B_{2n-1} \cos\left(\frac{(2n-1)\pi(x - S_\alpha + S_\beta)}{2S_\beta}\right) \exp\left[\left(-\frac{v}{2D} - \sqrt{\frac{v^2}{4D^2} + \left(\frac{(2n-1)\pi}{2S_\beta}\right)^2}\right) z\right]$$

with

$$B_{2n-1} = \left(-\frac{v}{2D} - \sqrt{\frac{v^2}{4D^2} + \left(\frac{(2n+1)\pi}{2S_\beta}\right)^2}\right)^{-1}$$

rapid solidification

$$\frac{2}{S_\beta} \int_{S_\alpha - S_\beta}^{S_\alpha} f_\beta(x) \cos\left(\frac{(2n+1)\pi(x - S_\alpha + S_\beta)}{2S_\beta}\right) dx$$

Solution to the diffusion equation

b/ for the β phase lamella $x \in \left[S_\alpha, S_\alpha + S_\beta\right]$ $z \geq 0$

$$\delta C(x,z) = \sum_{n=1}^{\infty} B_{2n-1} \cos\left(\frac{(2n-1)\pi(x - S_\alpha + S_\beta)}{2S_\beta}\right)\exp\left(-\frac{(2n-1)\pi}{2S_\beta}z\right)$$

$$B_{2n-1} = -\frac{4}{(2n-1)\pi}\int_{S_\alpha - S_\beta}^{S_\alpha} f_\beta(x)\cos\left(\frac{(2n-1)\pi(x - S_\alpha + S_\beta)}{2S_\beta}\right)dx$$

$n = 1, 2, \ldots$

and

$$\left.\frac{\partial \delta C(x,z)}{\partial z}\right|_{z=0} = f_\beta(x), \quad x \in [0, S_\beta]$$

slow solidification

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Presentation of solution to the diffusion equation

a/ for the α phase lamella

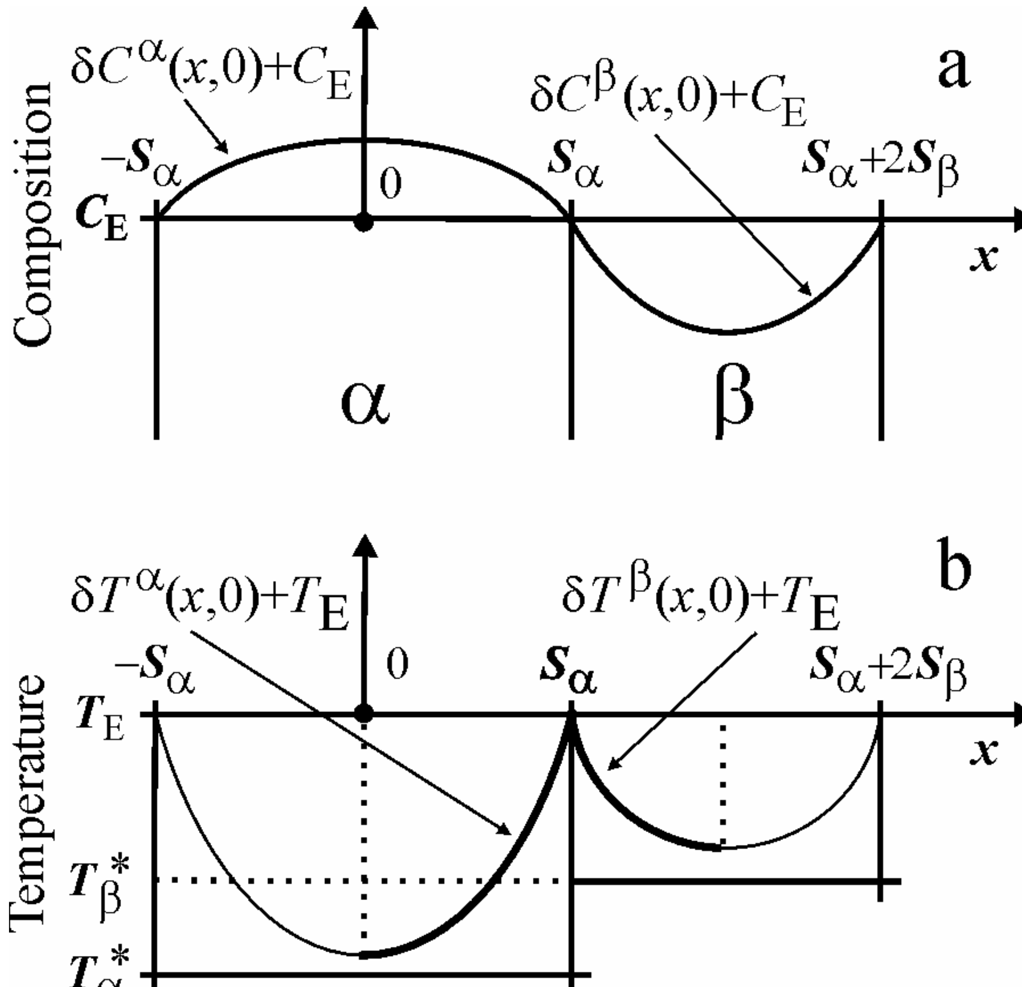
$x \in [0, S_\alpha]$ $z = 0$

b/ for the β phase lamella

$x \in [S_\alpha, S_\alpha + S_\beta]$ $z = 0$

FIG. 16

micro-field
of solute concentration
undercooling



Total mass balance within micro-field of solute concentration

$$\int_0^{\infty}\int_0^{S_\alpha}\delta C(x,z)dxdz+\int_0^{\infty}\int_{S_\alpha}^{S_\alpha+S_\beta}\delta C(x,z)dxdz+=\sum_{n=1}^{\infty}\frac{4(-1)^{n-1}D}{(2n-1)\pi}$$

$$\left(\frac{A_{2n-1}\,S_\alpha^2}{vS_\alpha+\sqrt{v^2S_\alpha^2+(2n-1)^2D^2\pi^2}}-\frac{B_{2n-1}\,S_\beta^2}{vS_\beta+\sqrt{v^2S_\beta^2+(2n-1)^2D^2\pi^2}}\right)=0$$

where

$$B_{2n-1}=\frac{A_{2n-1}\,S_\alpha^2\left(vS_\beta+\sqrt{v^2S_\beta^2+(2n-1)^2D^2\pi^2}\right)}{S_\beta^2\left(vS_\alpha+\sqrt{v^2S_\alpha^2+(2n-1)^2D^2\pi^2}\right)},\quad n=1,2,\ldots$$

for rapid solidification ↑

for slow solidification →

$$B_{2n-1}=A_{2n-1}\left(\frac{S_\alpha}{S_\beta}\right)^2,\quad n=1,2,\ldots$$

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Visualization of total mass balance within micro-field of solute concentration

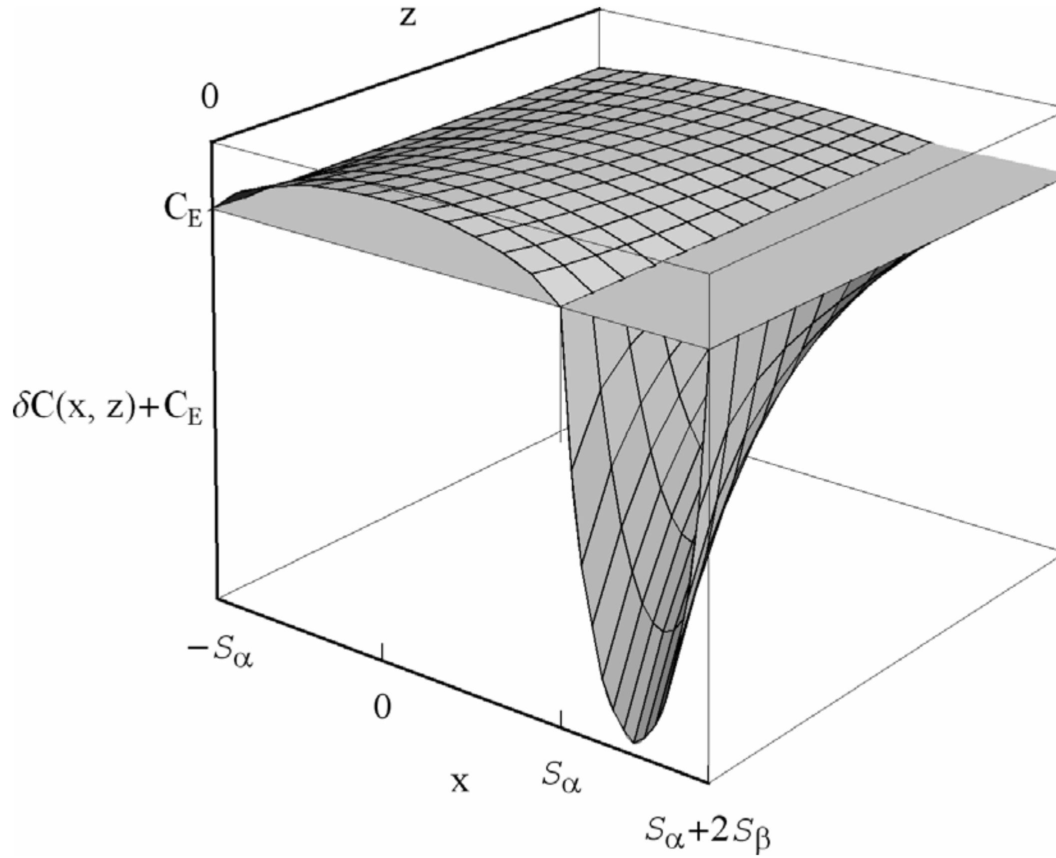


FIG. 17

mass balance calculated for planar solid / liquid interface

Local mass balance within micro-field of solute concentration

local mass balance is satisfied at z = 0 for α phase lamella and z = d for β phase lamella

$$\int_0^{S_\alpha} \delta C(x,0)\,dx + \int_{S_\alpha}^{S_\alpha+S_\beta} \delta C(x,d)\,dx = 0$$

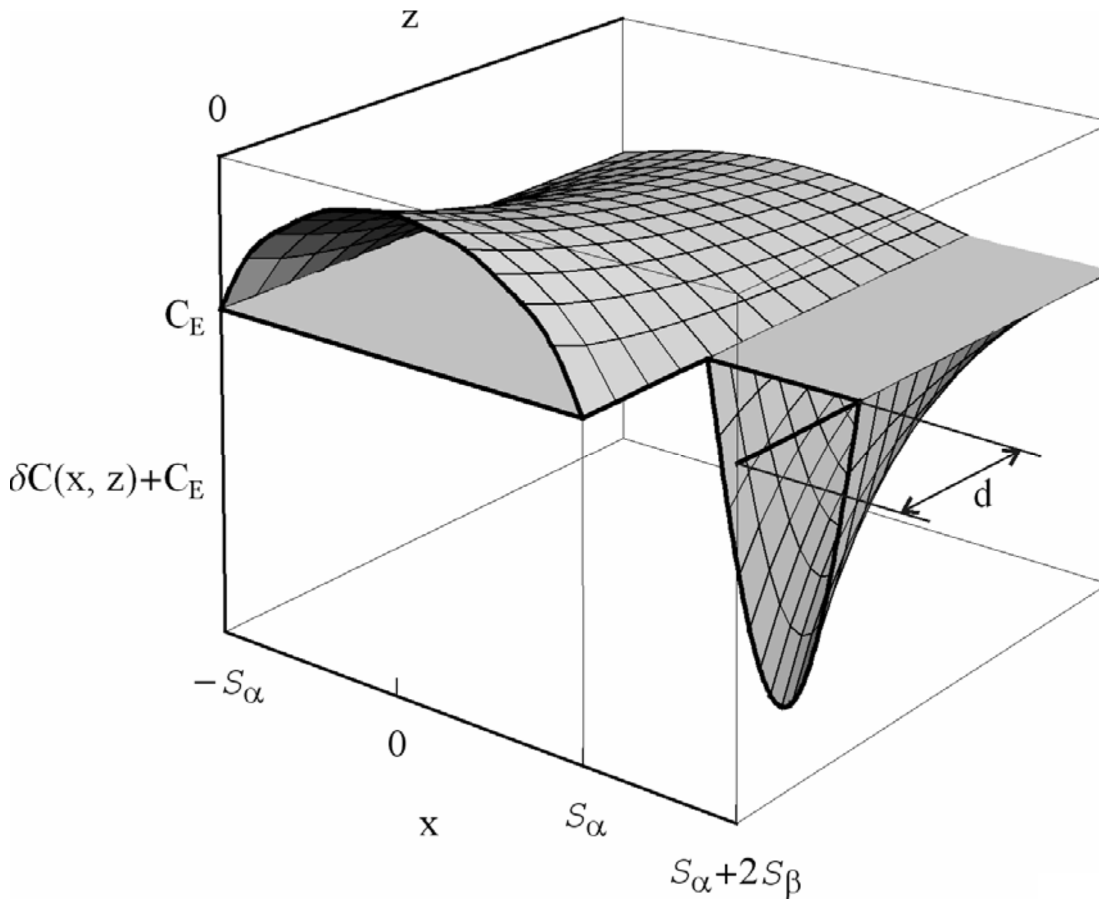
after some rearrangements

$$\sum_{n=1}^{\infty} A_{2n-1} \frac{2S_\alpha(-1)^{n-1}}{(2n-1)\pi} - \sum_{n=1}^{\infty} B_{2n-1} \frac{2S_\beta(-1)^{n-1}}{(2n-1)\pi} \exp\left(-\frac{vS_\beta + \sqrt{v^2 S_\beta^2 + (2n-1)^2 D^2 \pi^2}}{2DS_\beta} d \right) = 0$$

d - phase protrusion

Visualization of local mass balance within micro-field of solute concentration

FIG. 18



protrusion, d,
is the transient phase;
it has property of the liquid
but structure of the solid

Definition of the phase protrusion

rapid solidification

$$\sum_{n=1}^{\infty} A_{2n-1} \frac{(-1)^{n-1}}{(2n-1)} \times$$
$$\left(1 - \frac{S_\alpha \left(vS_\beta + \sqrt{v^2 S_\beta^2 + (2n-1)^2 D^2 \pi^2}\right)}{S_\beta \left(vS_\alpha + \sqrt{v^2 S_\alpha^2 + (2n-1)^2 D^2 \pi^2}\right)} \exp\left(-\frac{vS_\beta + \sqrt{v^2 S_\beta^2 + (2n-1)^2 D^2 \pi^2}}{2DS_\beta} d\right)\right)$$
$$= 0$$

slow solidification

$$\sum_{n=1}^{\infty} A_{2n-1} \frac{(-1)^{n-1}}{(2n-1)} \left(1 - \frac{S_\alpha}{S_\beta} \exp\left(-\frac{(2n-1)\pi}{2S_\beta} d\right)\right) = 0$$

Confirmation for the existence of the phase protrusion

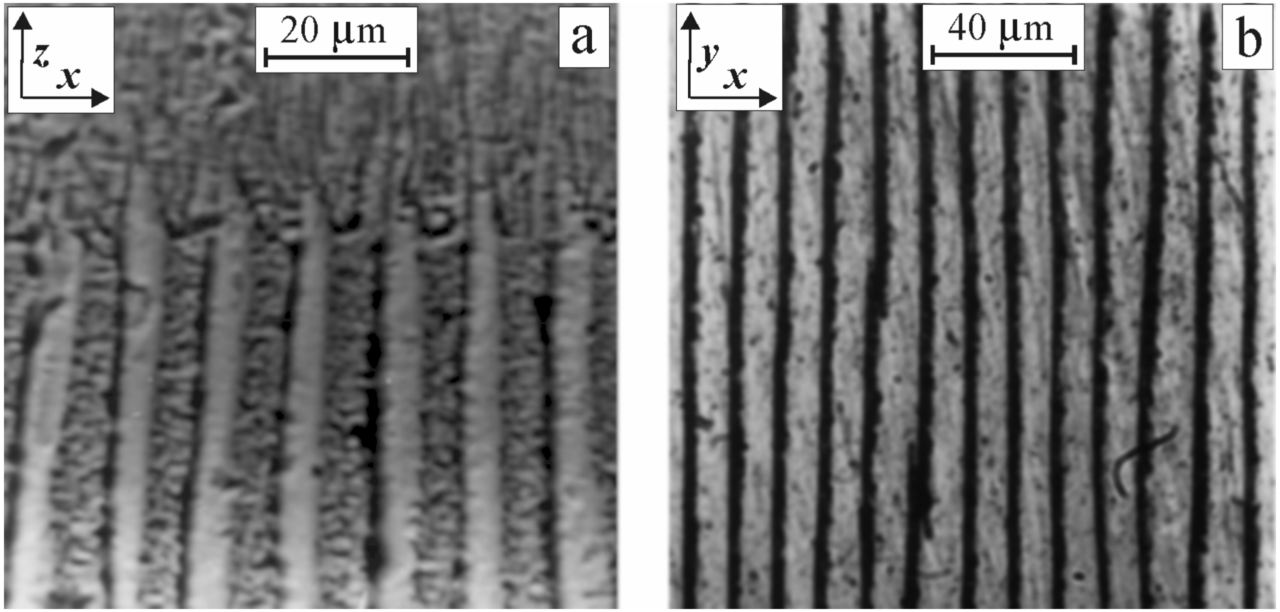


FIG. 19

oriented growth of
the (Pb) – (Cd) composite *in situ*

phase protrusion visible for (Cd) - leading phase, FIG. 19a

α / β inter-phase mass balance

the s / l interface mass balance requires

$$S_\alpha \frac{\partial \delta C^\alpha(x,0)}{\partial z} = S_\alpha \frac{v}{D}(1-k_\alpha)C^\alpha(x,0) \qquad x \in [0, S_\alpha]$$

$$S_\beta \frac{\partial \delta C^\beta(x,d)}{\partial z} = S_\beta \frac{v}{D}(1-k_\beta)C^\beta(x,d) \qquad x \in [S_\alpha, S_\alpha + S_\beta]$$

mass balance at the α / β inter-phase

$$\lim_{x \to S_\alpha^-} S_\alpha \frac{\partial \delta C^\alpha(x,0)}{\partial z} + \lim_{x \to S_\alpha^+} S_\beta \frac{\partial \delta C^\beta(x,d)}{\partial z} =$$

$$S_\alpha \frac{v}{D} C_0^\alpha(S_\alpha,0) + S_\beta \frac{v}{D} C_0^\beta(S_\alpha,d) = \frac{v}{D}\left(S_\alpha C_0^\alpha(S_\alpha,0) + S_\beta C_0^\beta(S_\alpha,d)\right) = 0$$

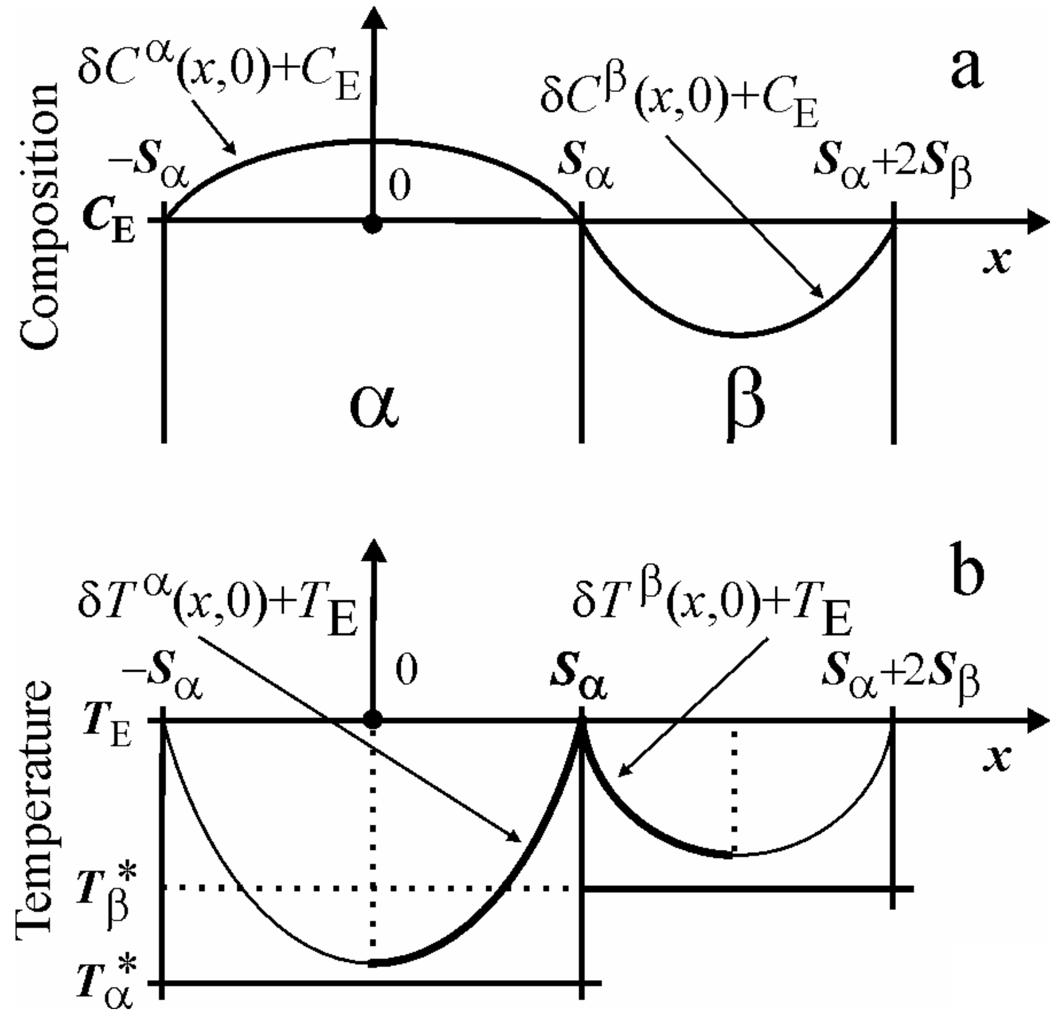
Triple point of the s / l interface

not only mechanical equilibrium but thermodynamic equilibrium at the triple point is satisfied as well

solute concentration micro-field exists within the transition phase, d, (over leading phase) as if it was the liquid phase

solute concentration undercooling

FIG. 20



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Concluding remarks

at slow solidification, typical for composite *in situ* growth

$$B_{2n-1} = A_{2n-1}\left(\frac{S_\alpha}{S_\beta}\right)^2, \quad n = 1,2,\ldots$$

$$\left| f_\beta\left(x\frac{S_\beta}{S_\alpha}\right)\right| = \left| f_\alpha(x)\right|\left(\frac{S_\alpha}{S_\beta}\right)^2$$

the current description of micro-filed of solute concentration
can be mathematically reduced
to the J-H equation, however under condition that phase diagram
would be symmetrical one
that is $C_0^\alpha = C_0^\beta$ *and consequentially* $S_\alpha = S_\beta$*, additionally,*
phase protrusion becomes $d = 0$*,*
C_E *comes back to the* α / β *inter-phase boundary*
and mass balance is satisfied at each co-ordinate, z

METRO

MEtallurgical TRaining On-line

Mass transport at the solid/liquid interface of growing composite *in situ*

End of the lecture

Education and Culture