

METRO  
MEtallurgical TRaining On-line



# Thermodynamic basis of modeling of phase change phenomena

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CzUT



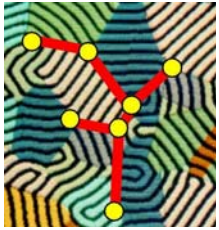
Education and Culture



# Plan of presentation



- Basic concepts
- Gibbs free energy
- Lever rule
- Cooling curves
- Binary phase equilibrium diagrams
- Ternary phase equilibrium diagrams



# Basic concepts



- Alloy
- Phase
- System
- Phase equilibrium
- Thermodynamic equilibrium



# Basic concepts



- Helmholtz free energy

$$F = E - TS$$

$E$  – internal energy,  $T$  - temperature,  $S$  - entropy

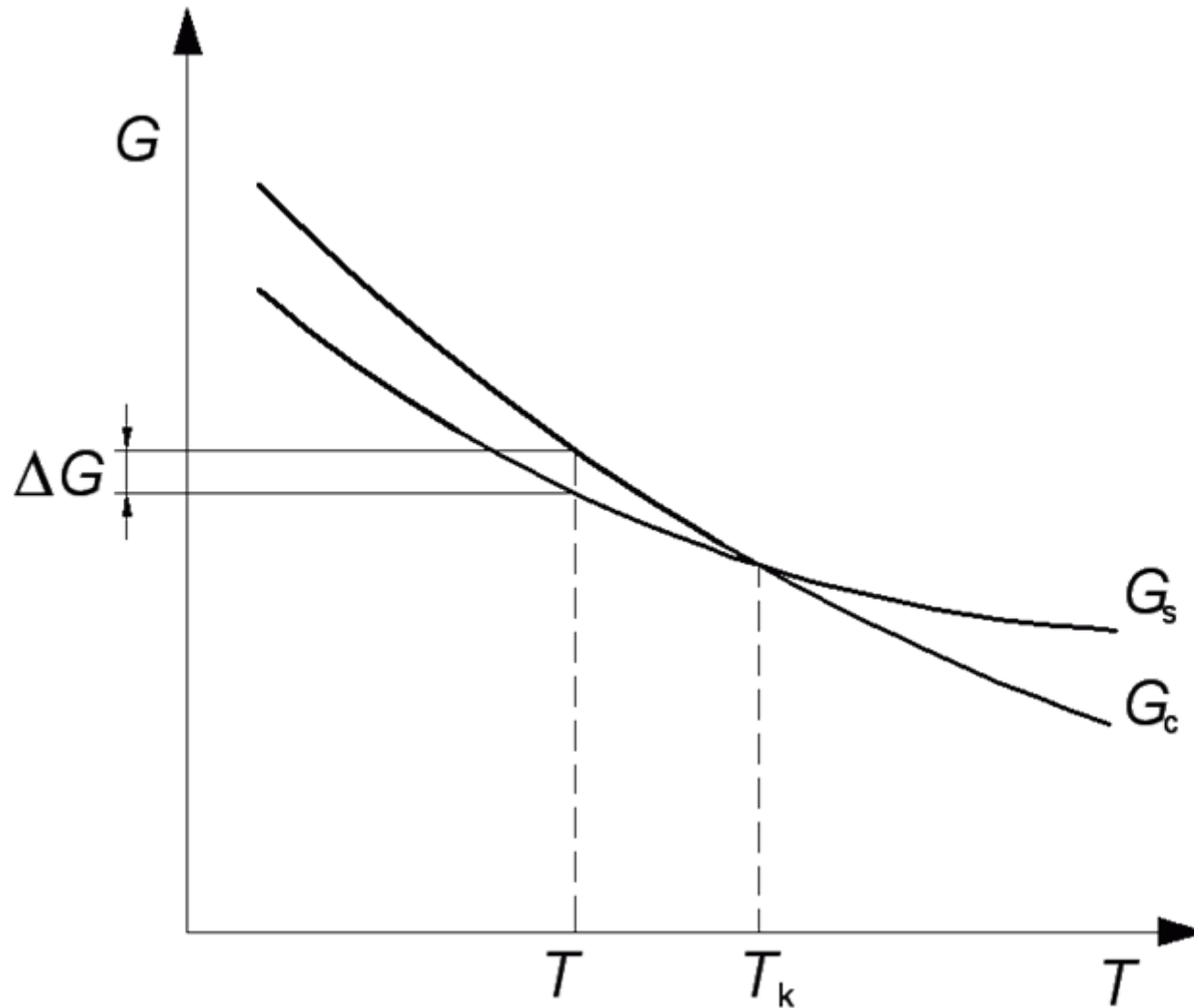
- Gibbs free energy

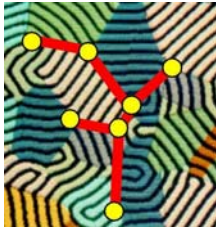
$$G = E + pV - TS$$

$p$  - pressure,  $V$  - volume



# Gibbs free energy





# Gibbs phase rule



Equation describing Gibbs phase rule:

$$S = n - c - f + p$$

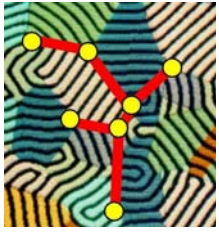
$S$  – number of degrees of freedom

$n$  – number of dependent components

$c$  - number of reversible chemical reactions

$f$  – number of phases

$p$  – number of physical factors affecting equilibrium



# Gibbs phase rule



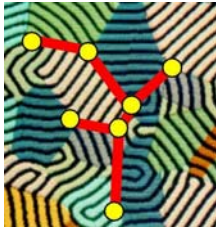
If there is no chemical or other reversible reactions between components and only physical factor which influences system is temperature, then phase rule is simplified to this equation:

$$S = n - f + 1$$

$S$  – number of degrees of freedom

$n$  – number of dependent components

$f$  – number of phases



# Newton law



Cooling is described by Newton law

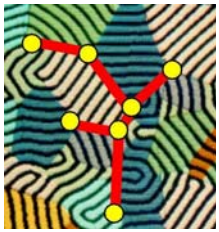
$$\Delta T = \Delta T_0 \cdot e^{-\alpha \tau}$$

$\Delta T_0$  — initial difference between temperature of heated body and surrounding environment (for time  $\tau = 0$ )

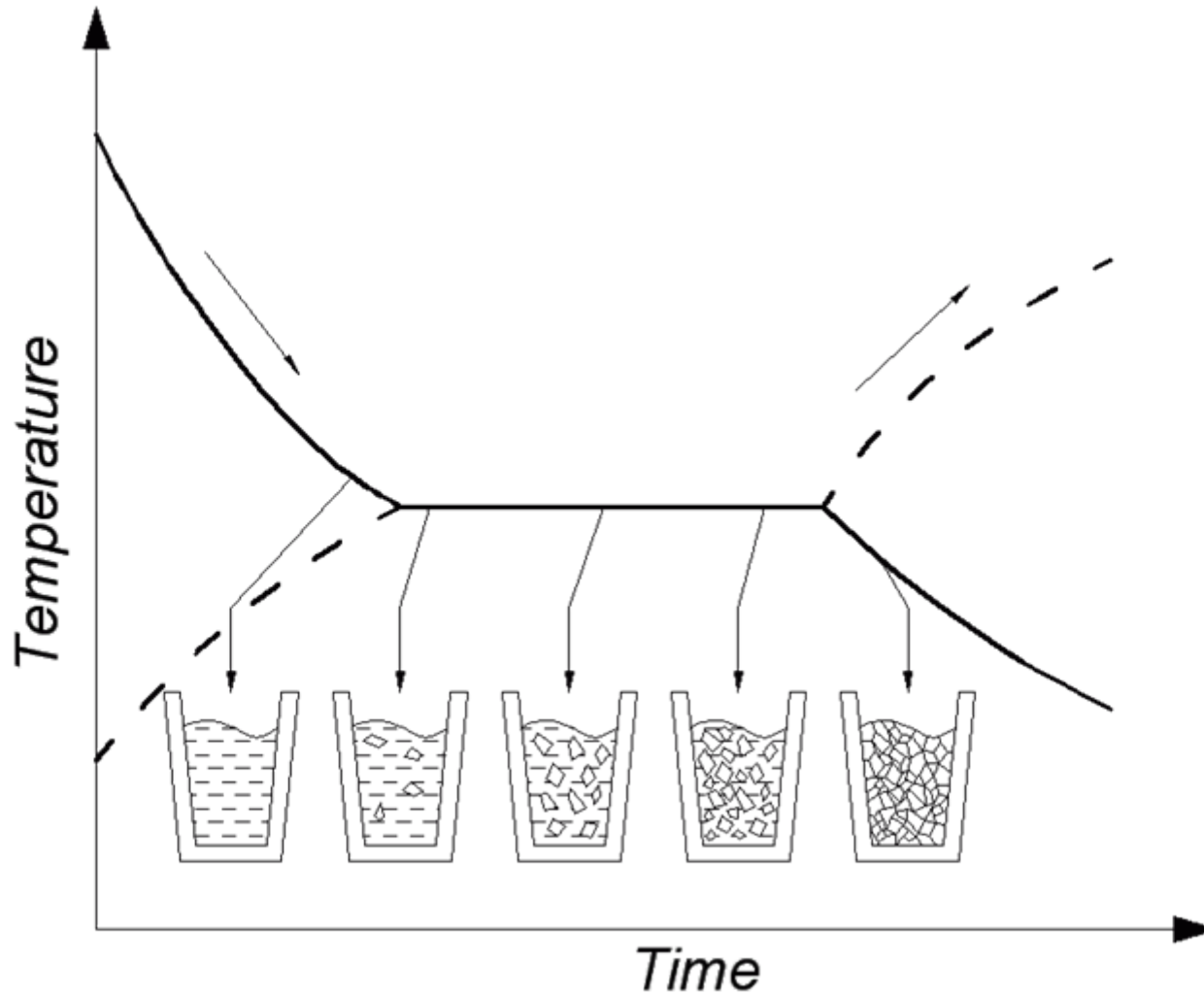
$\Delta T$  - denotes difference between temperature of heated body and ambient temperature for time  $\tau$

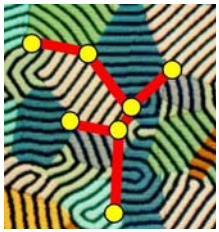
$\alpha$  – cooling factor



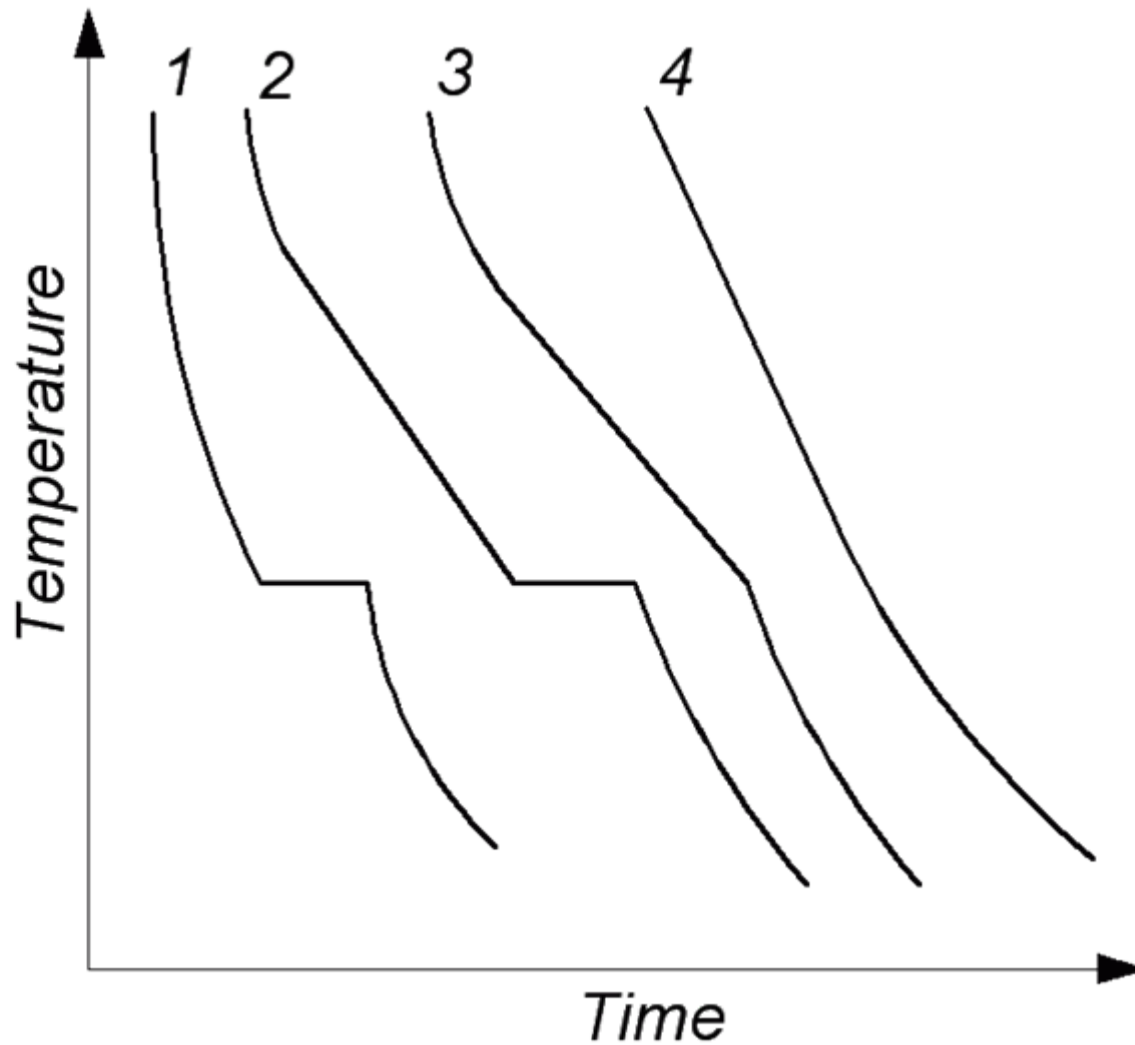


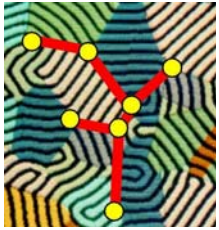
# Cooling curves



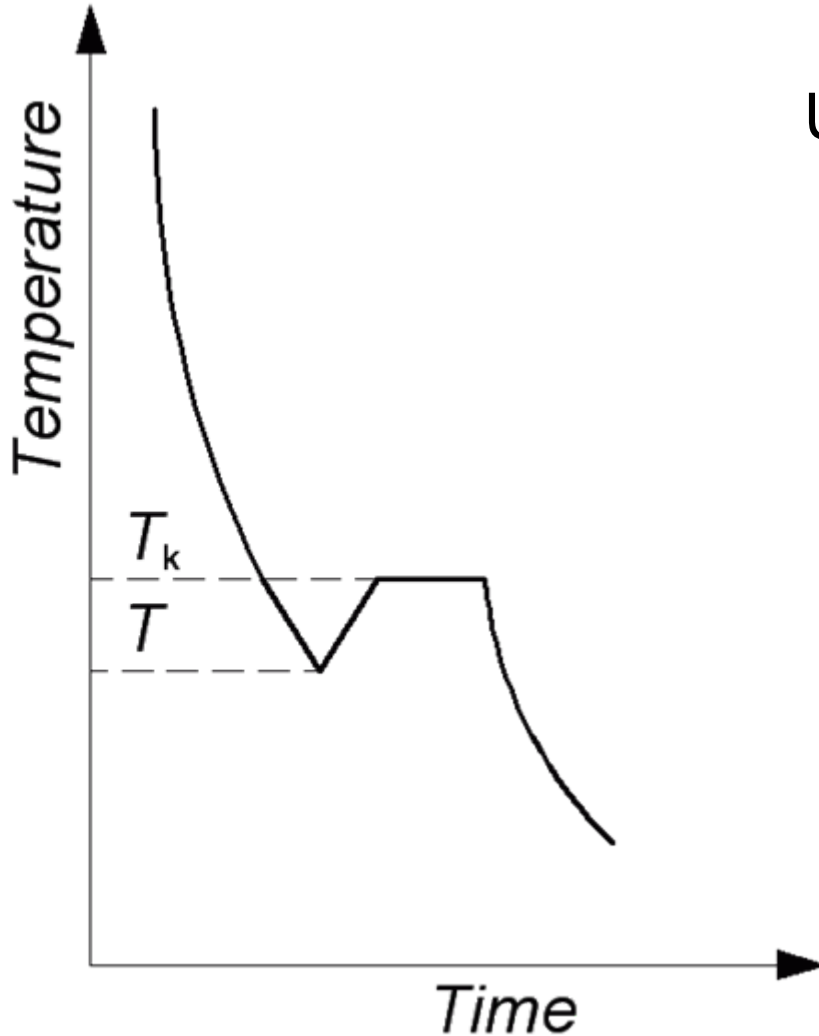


# Cooling curves



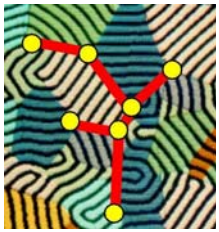


# Undercooling

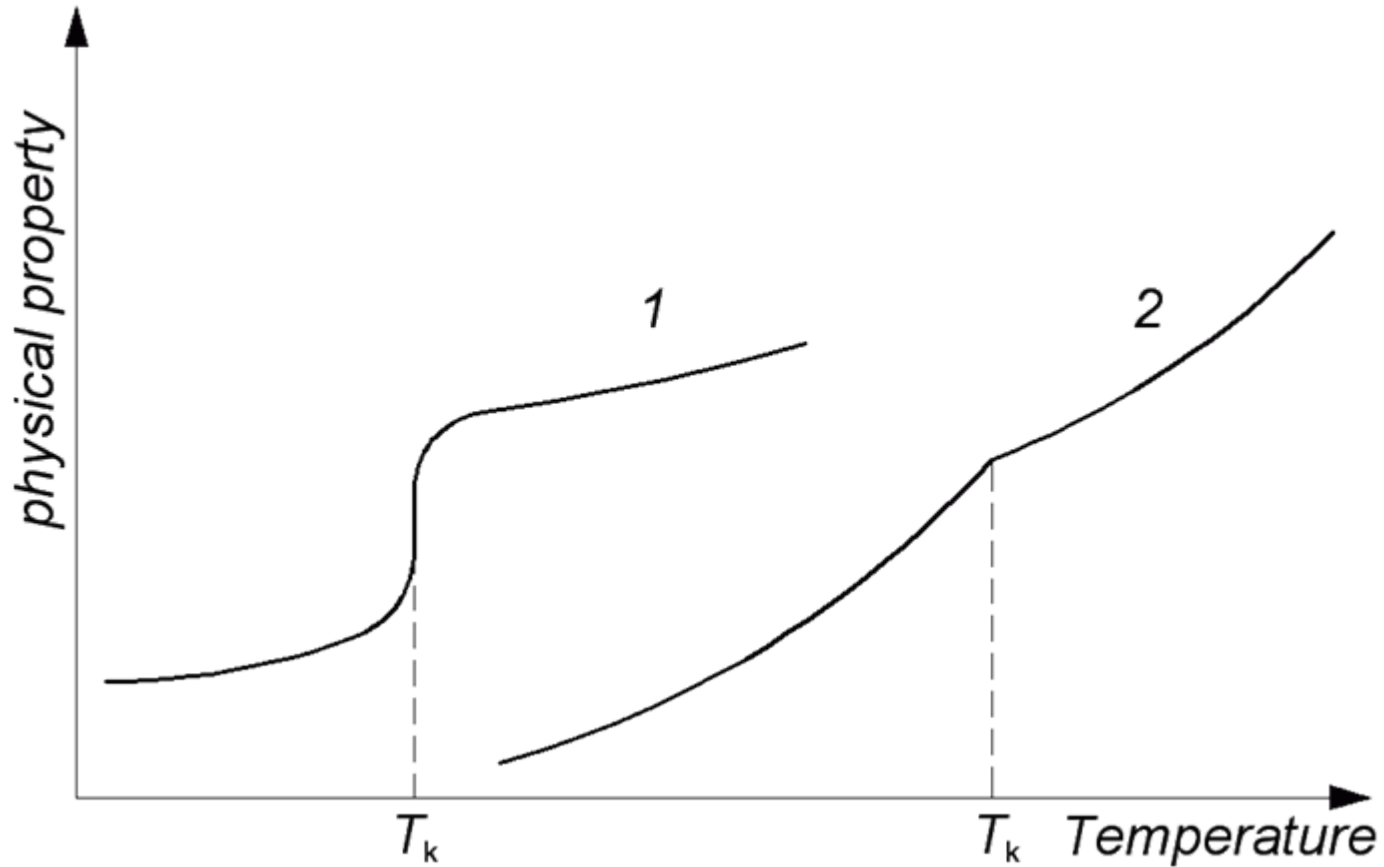


Undercooling:

$$\Delta T = T_K - T$$

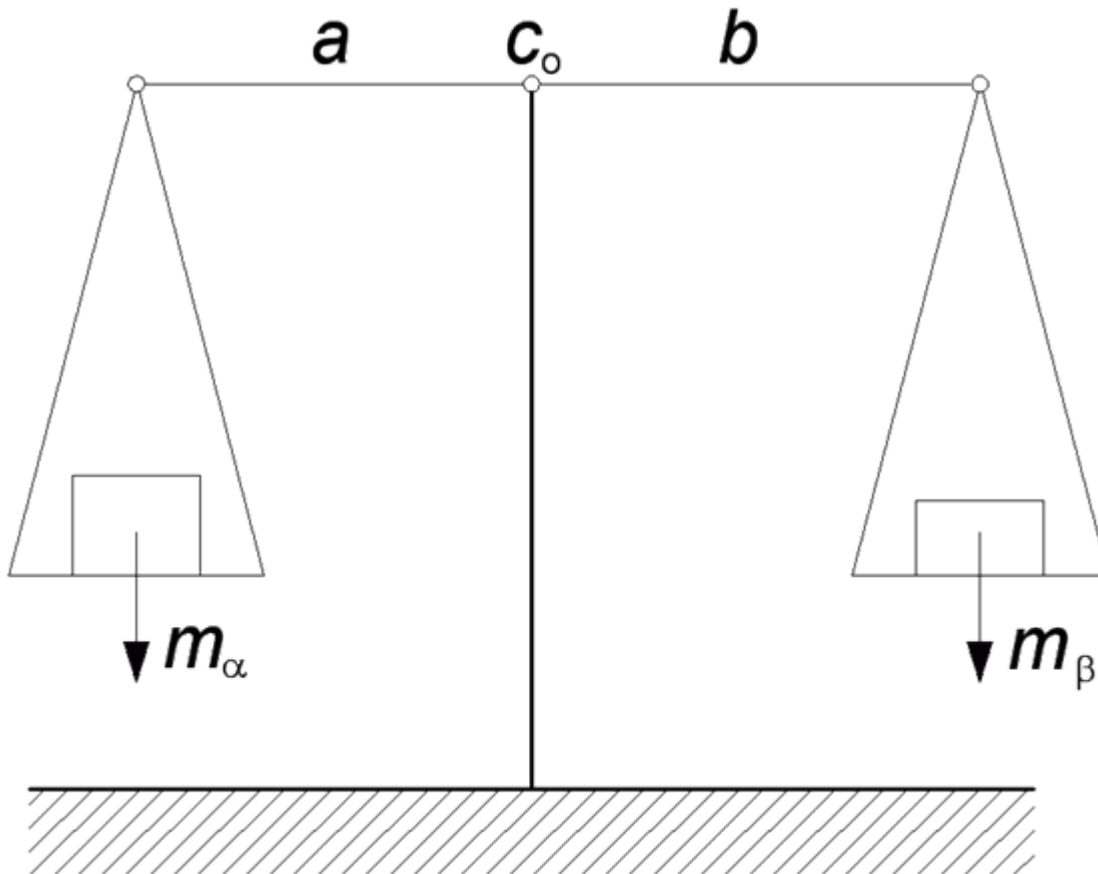


# Physical properties of cooled materials



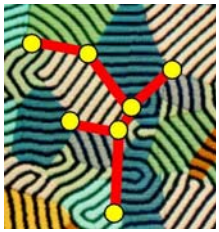


# Lever rule

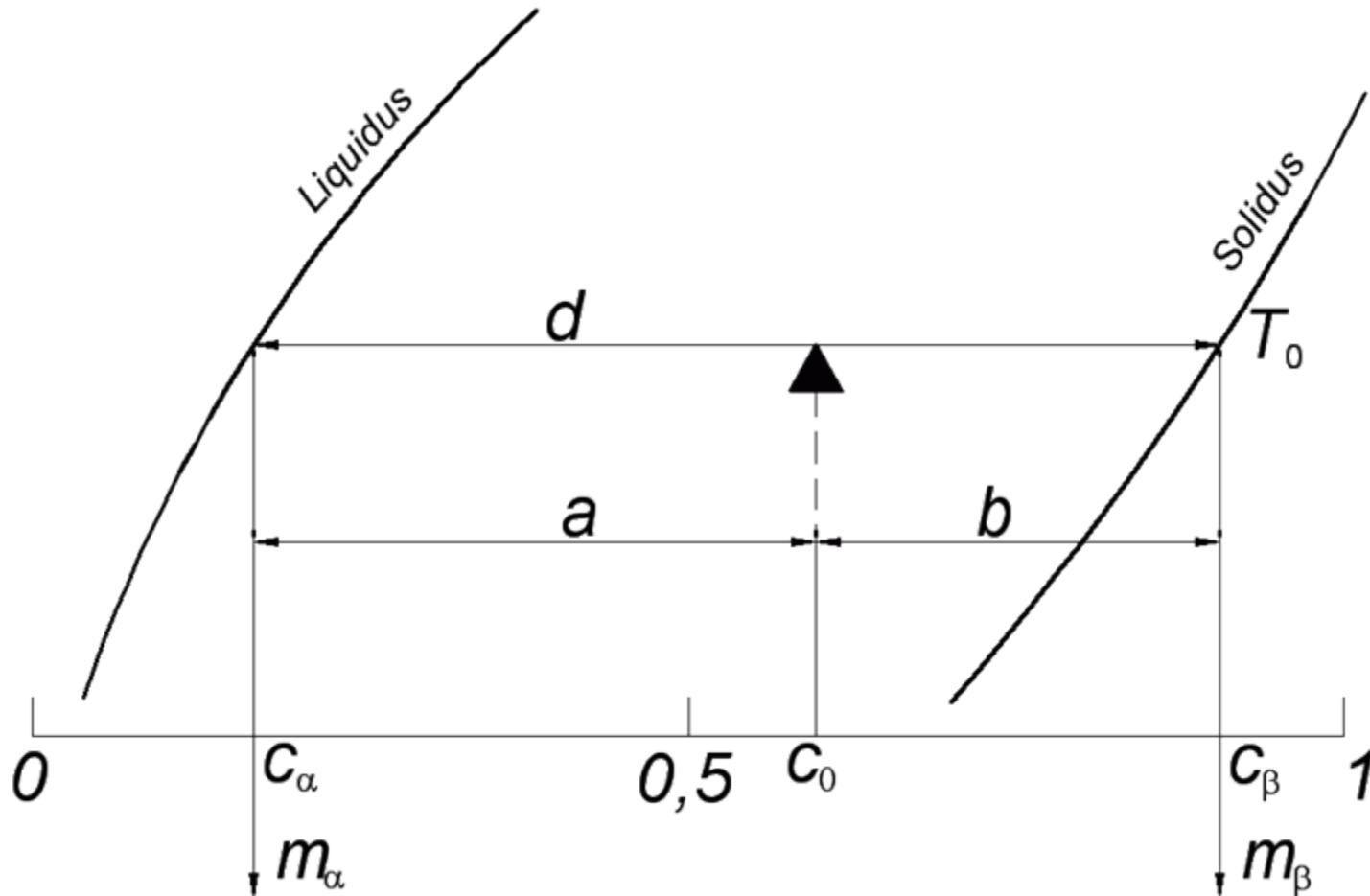


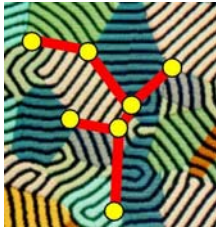
$$m_{\alpha} a = (1 - m_{\alpha}) b \quad (1)$$

$$\frac{m_{\alpha}}{1 - m_{\alpha}} = \frac{b}{a} \quad (2)$$



# Lever rule



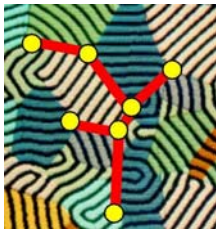


# Alloying phases



We can divide phases on

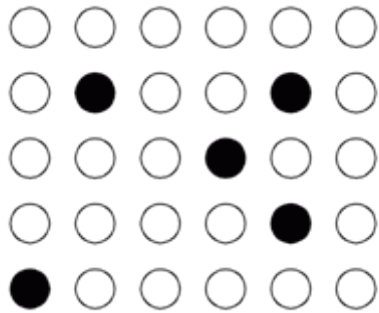
- Chemical elements
- Solid solutions
- Intermetallic phases



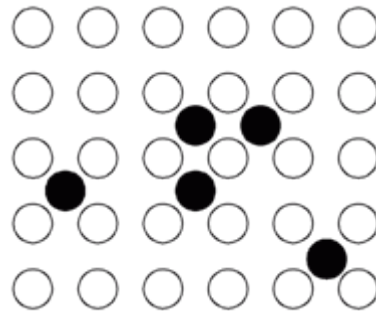
# Alloying phases



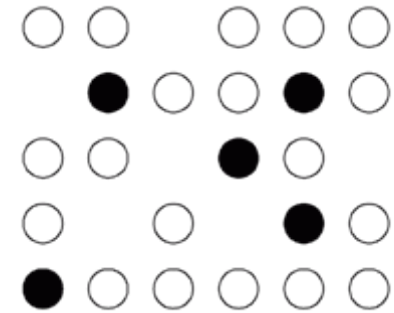
Solid solutions:



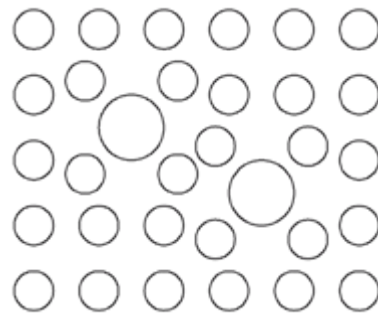
Picture 1



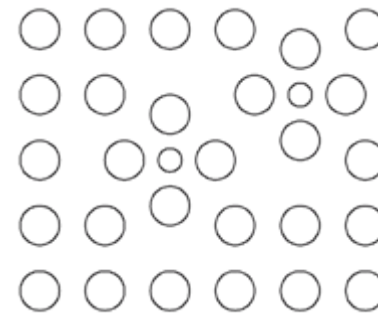
Picture 2



Picture 3

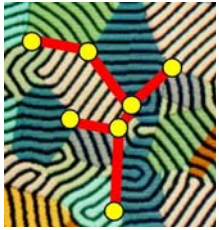


Picture 4



Picture 5





# Alloying phases



Intermetallic phases are characterized by:

- suitable crystal structure
- filling by atoms of chemical compounds typical for them positions in space lattice
- valence of components mostly runs away from valence in chemical conventional compounds
- the metallic bond

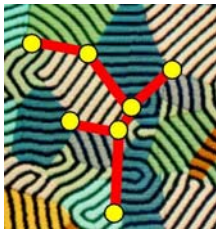


# Binary systems

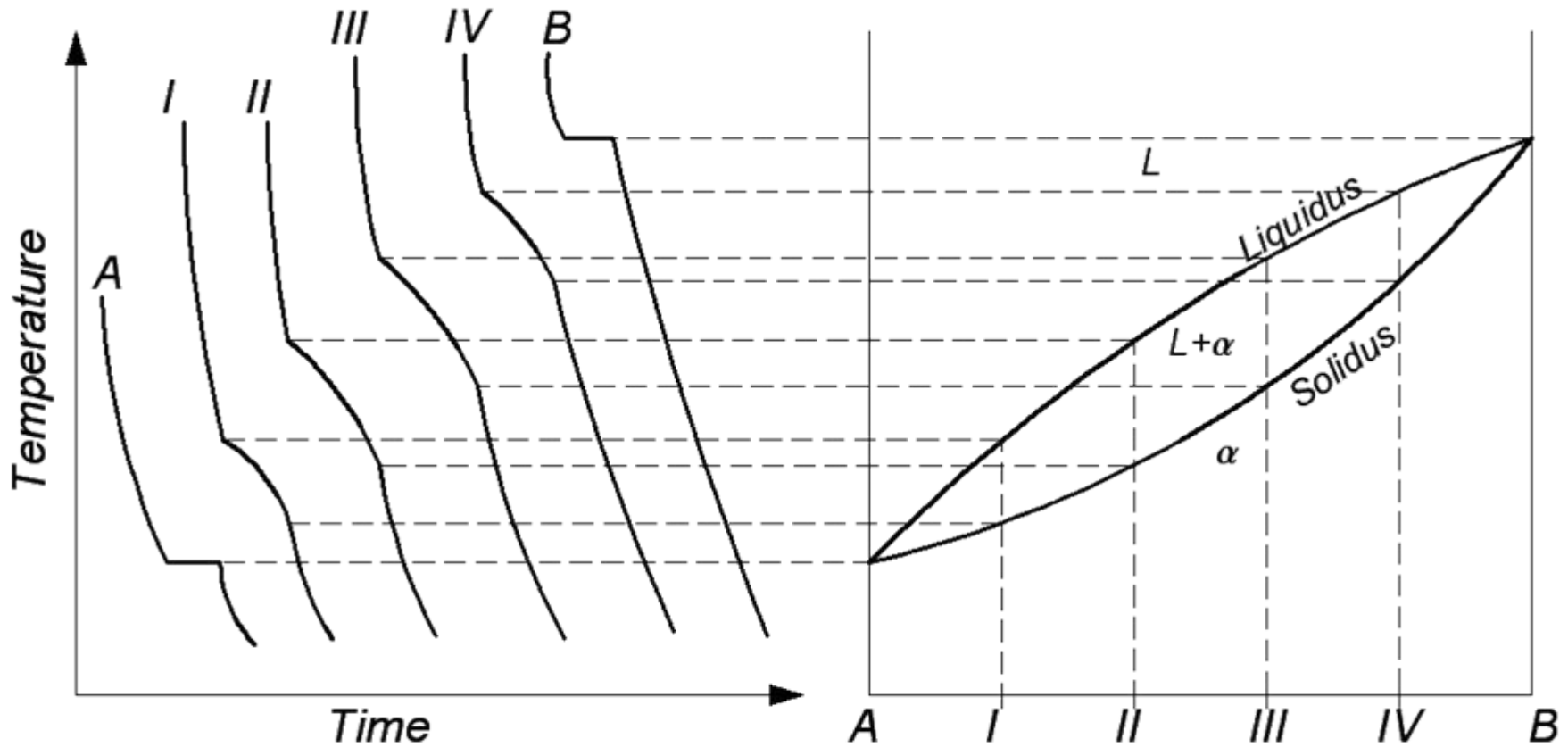


Hume-Rothery conditions:

- Similar type of crystal lattice of components
- Factor of atom size
- The close neighborhood of elements forming the solution in periodical system
- Effect of relative valence



# Binary systems – binary isomorphous system





# Binary systems – binary isomorphous system



- Gibbs phase rule for region  $L$  and region  $\alpha$ :

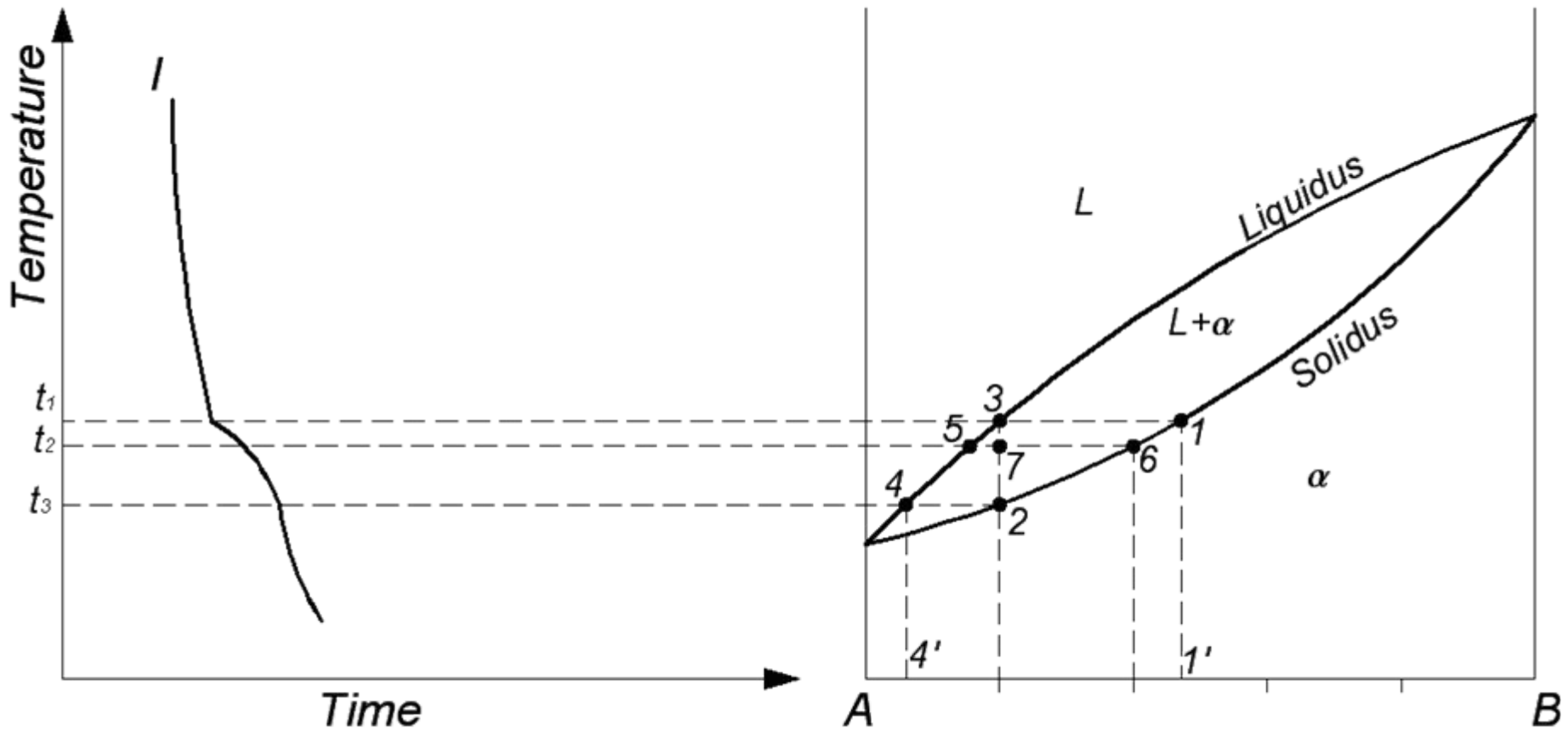
$$S = n - f + 1 = 2 - 1 + 1 = 2$$

- Gibbs phase rule for region  $L + \alpha$ :

$$S = n - f + 1 = 2 - 2 + 1 = 1$$



# Binary systems – binary isomorphous system

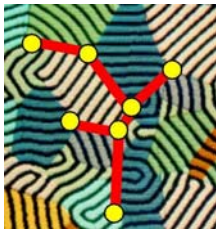




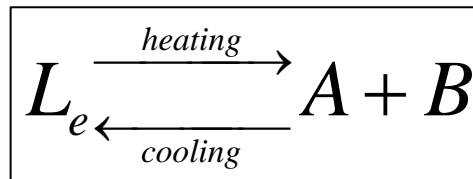
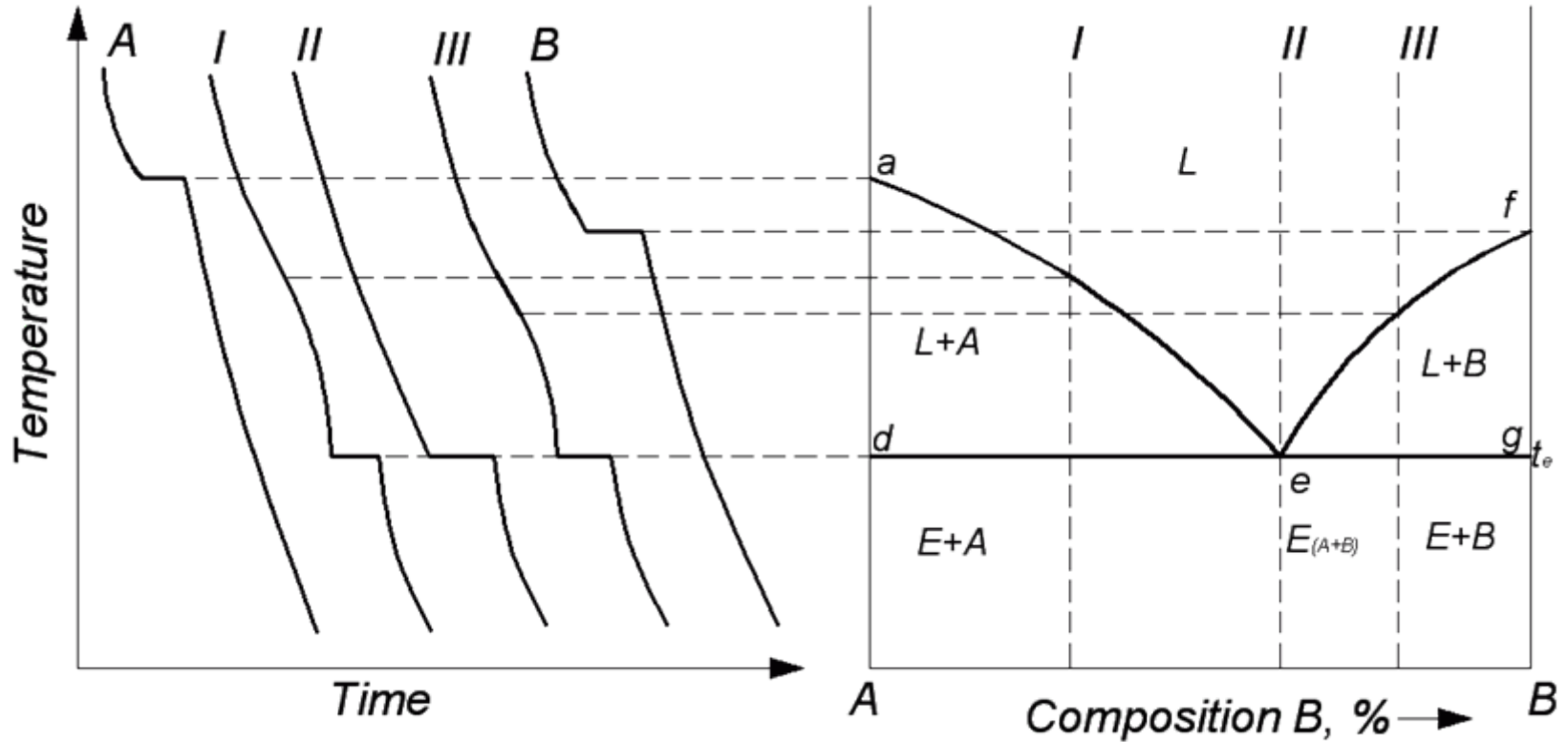
# Binary systems

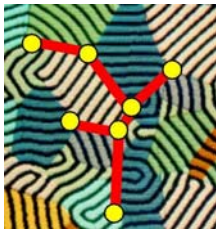


- Systems with no solubility in solid state (system with eutectic transition)
- Eutectic transition is characterized by:
  - It occurs in fixed temperature
  - Composition of liquid from which eutectics is formed is always the same
  - Relation of phases in eutectics is constant

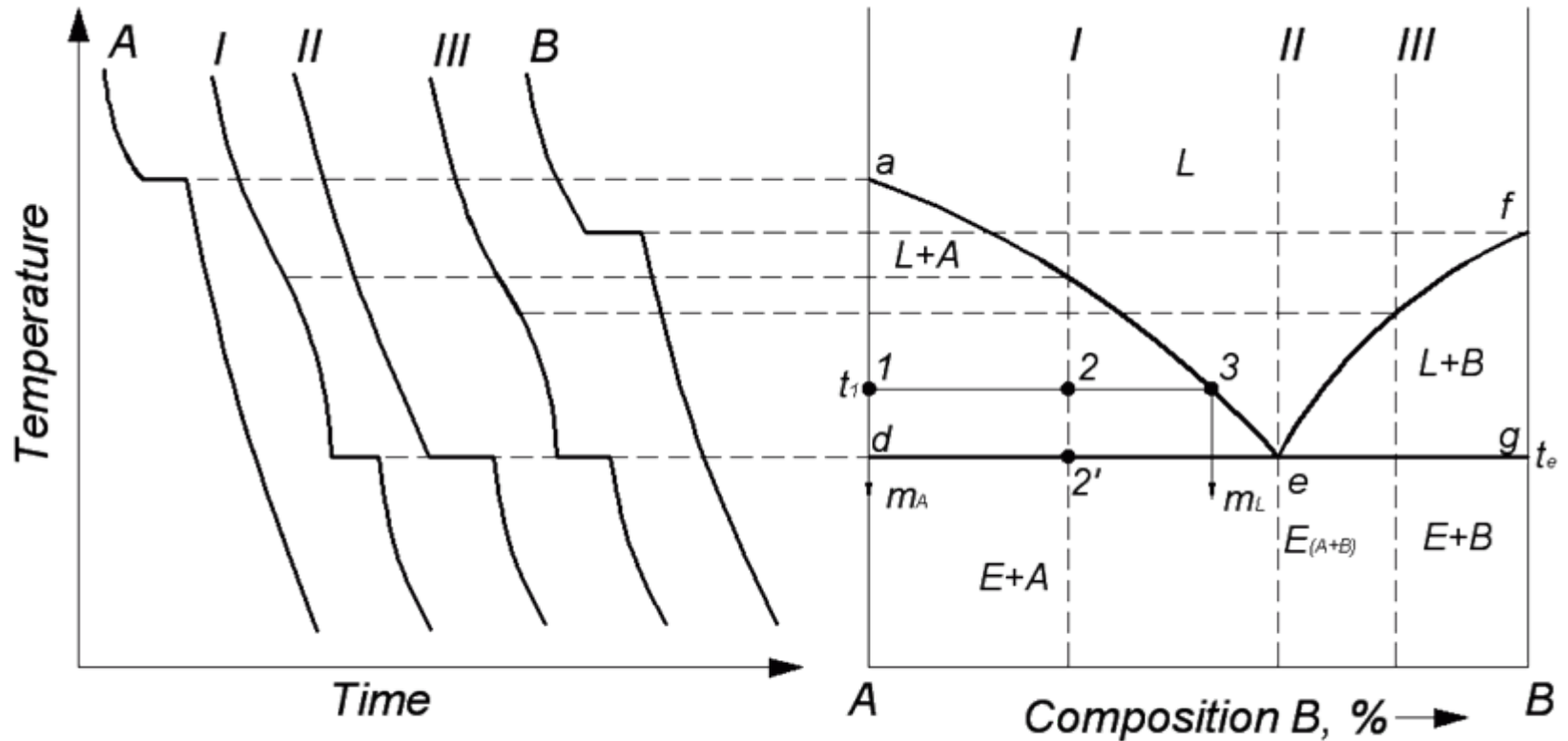


# Binary systems with no solid solubility – eutectic system

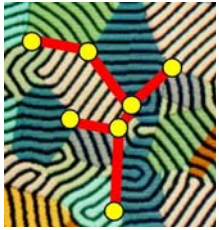




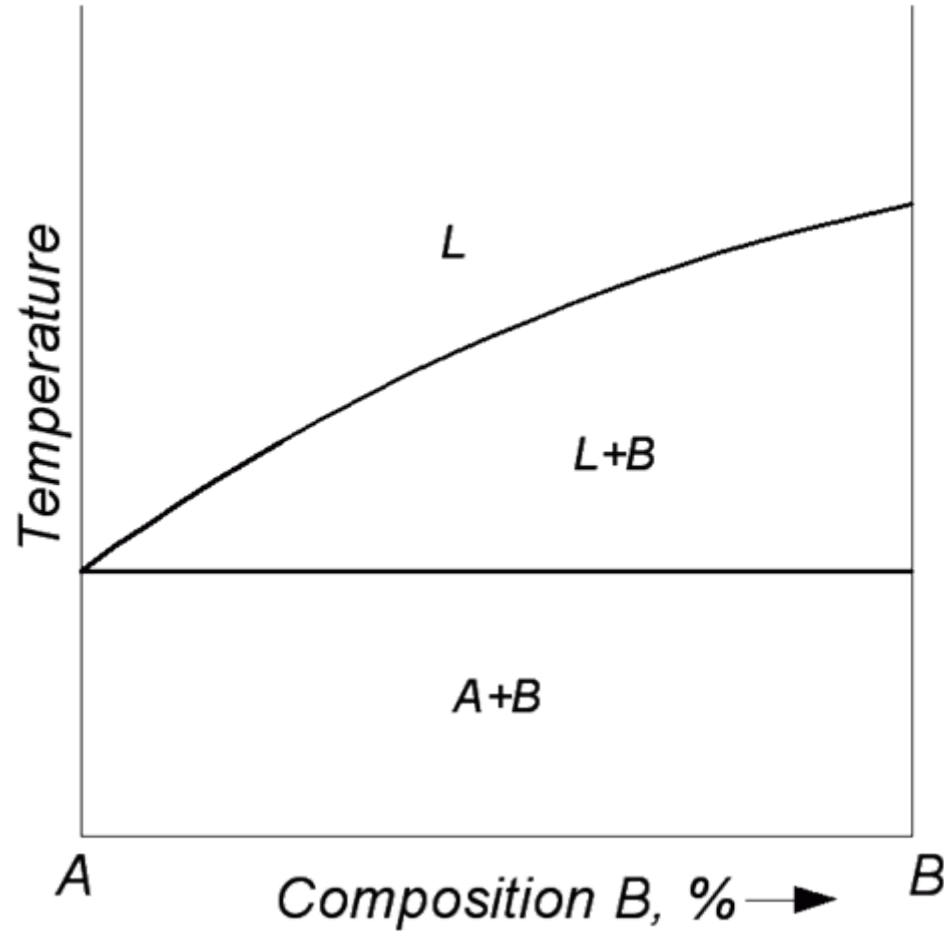
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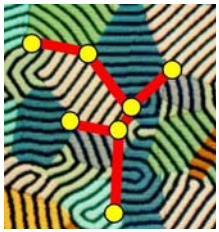




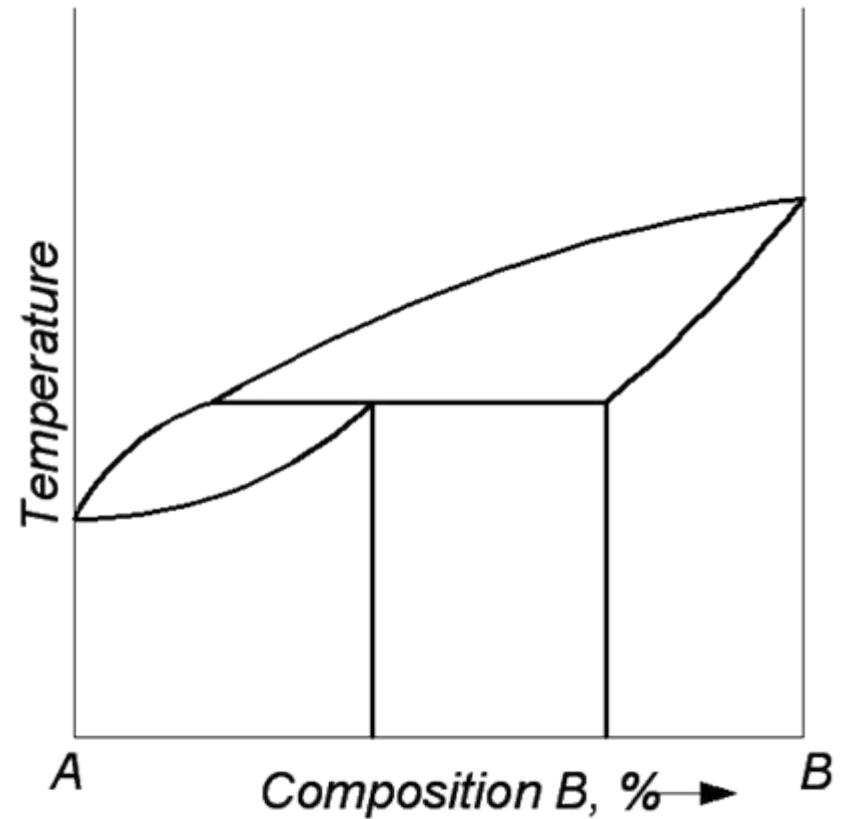
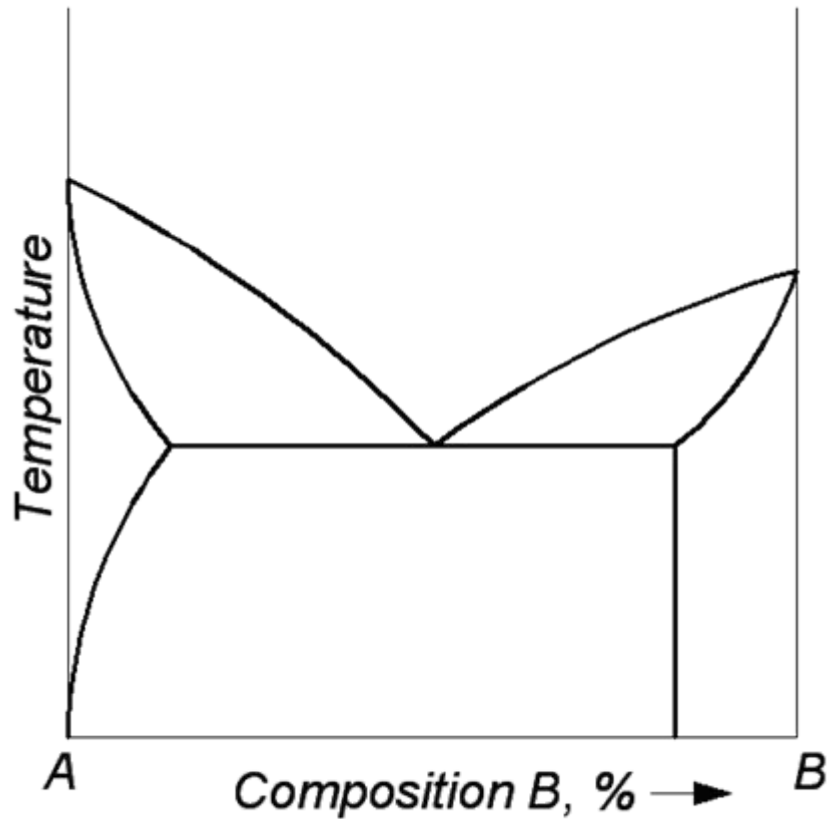


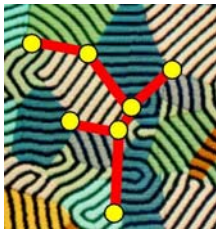
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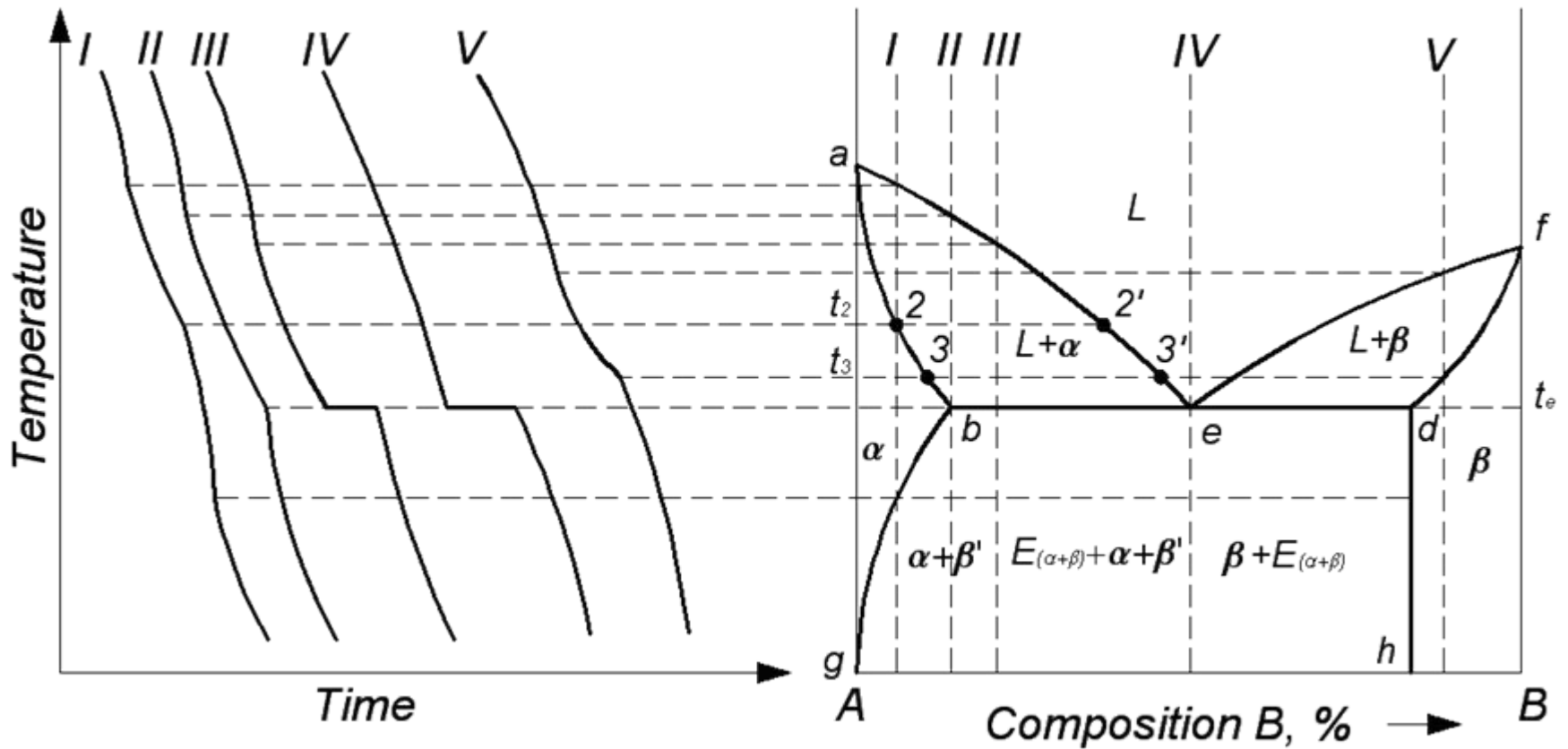


# Binary systems with limited solid solubility



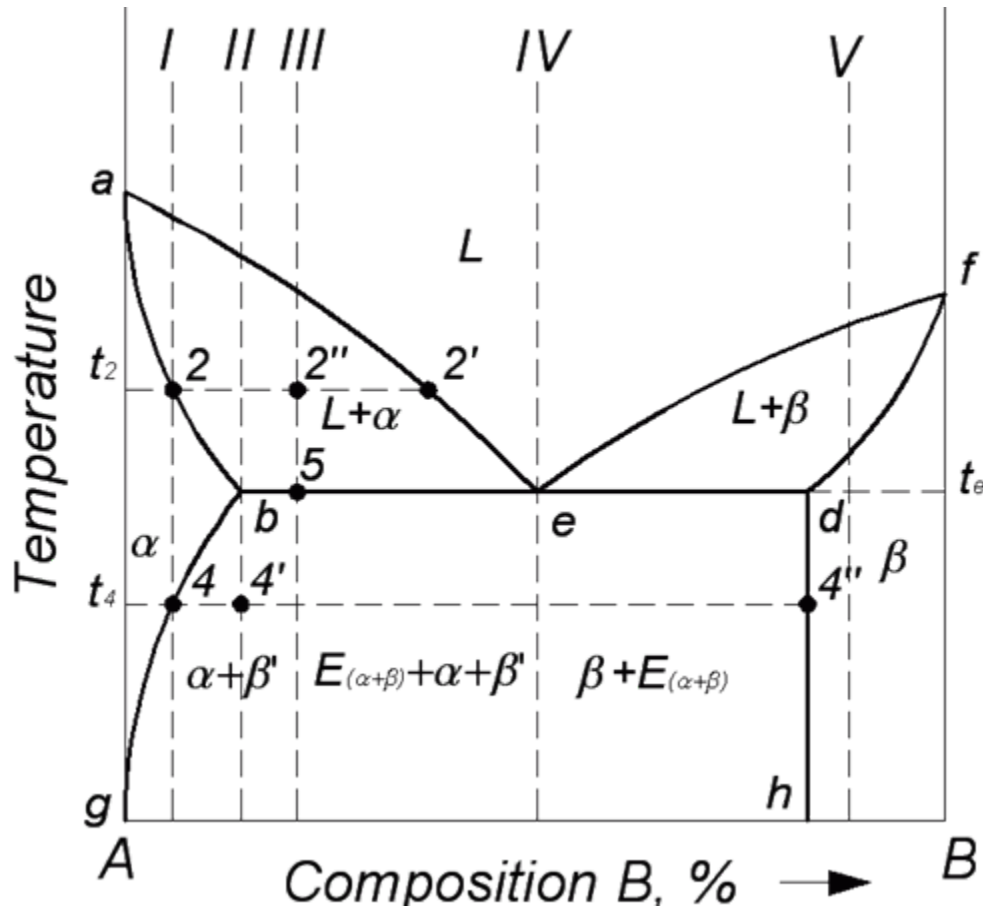


# Binary systems with limited solid solubility – eutectic system



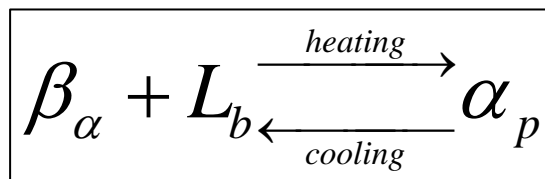
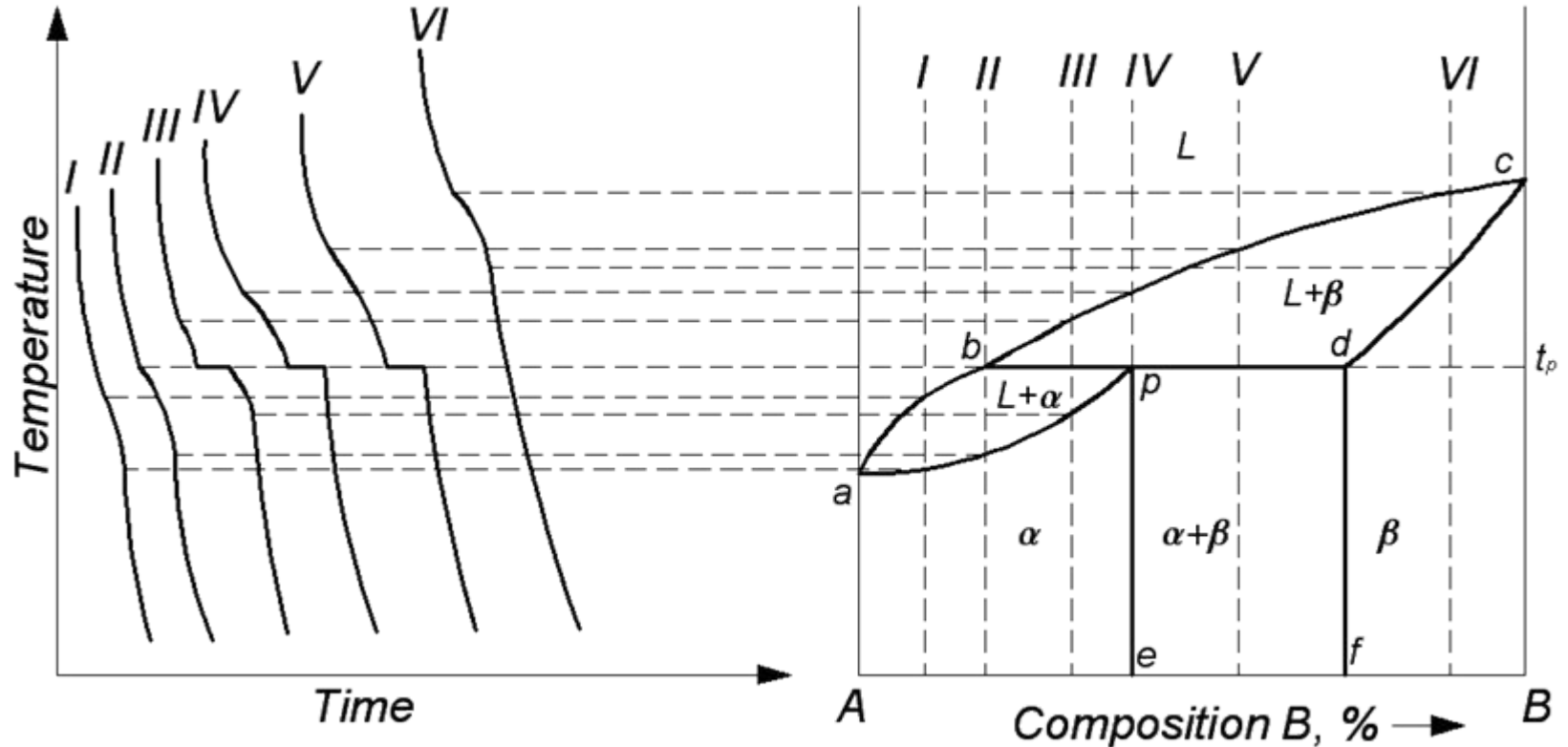


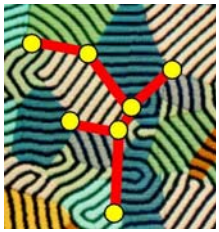
# Binary systems with limited solid solubility – eutectic system



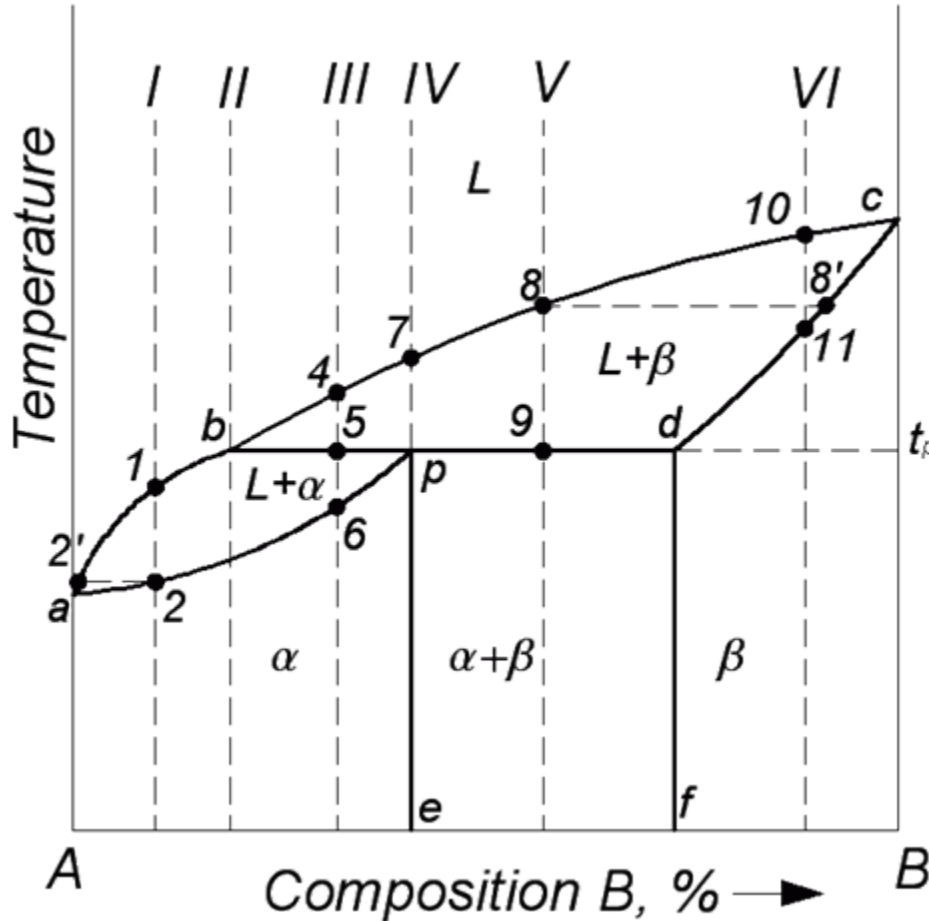


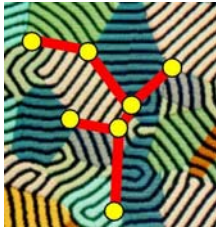
# Binary systems with limited solid solubility – peritectic system



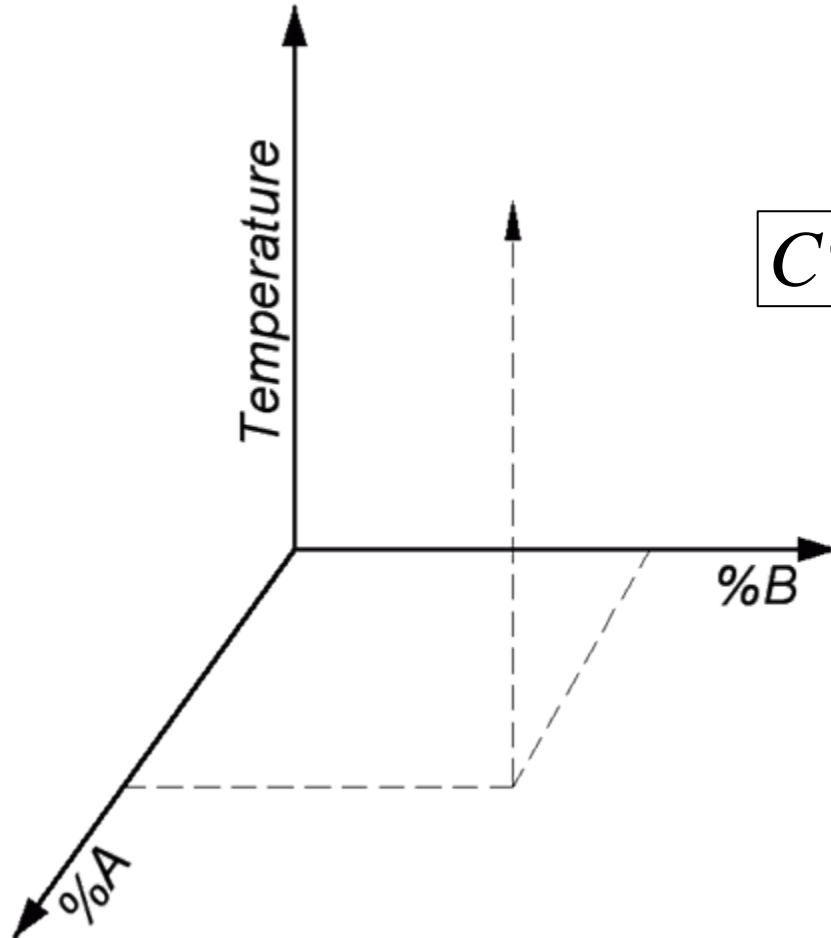


# Binary systems with limited solid solubility – peritectic system

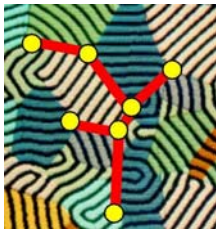




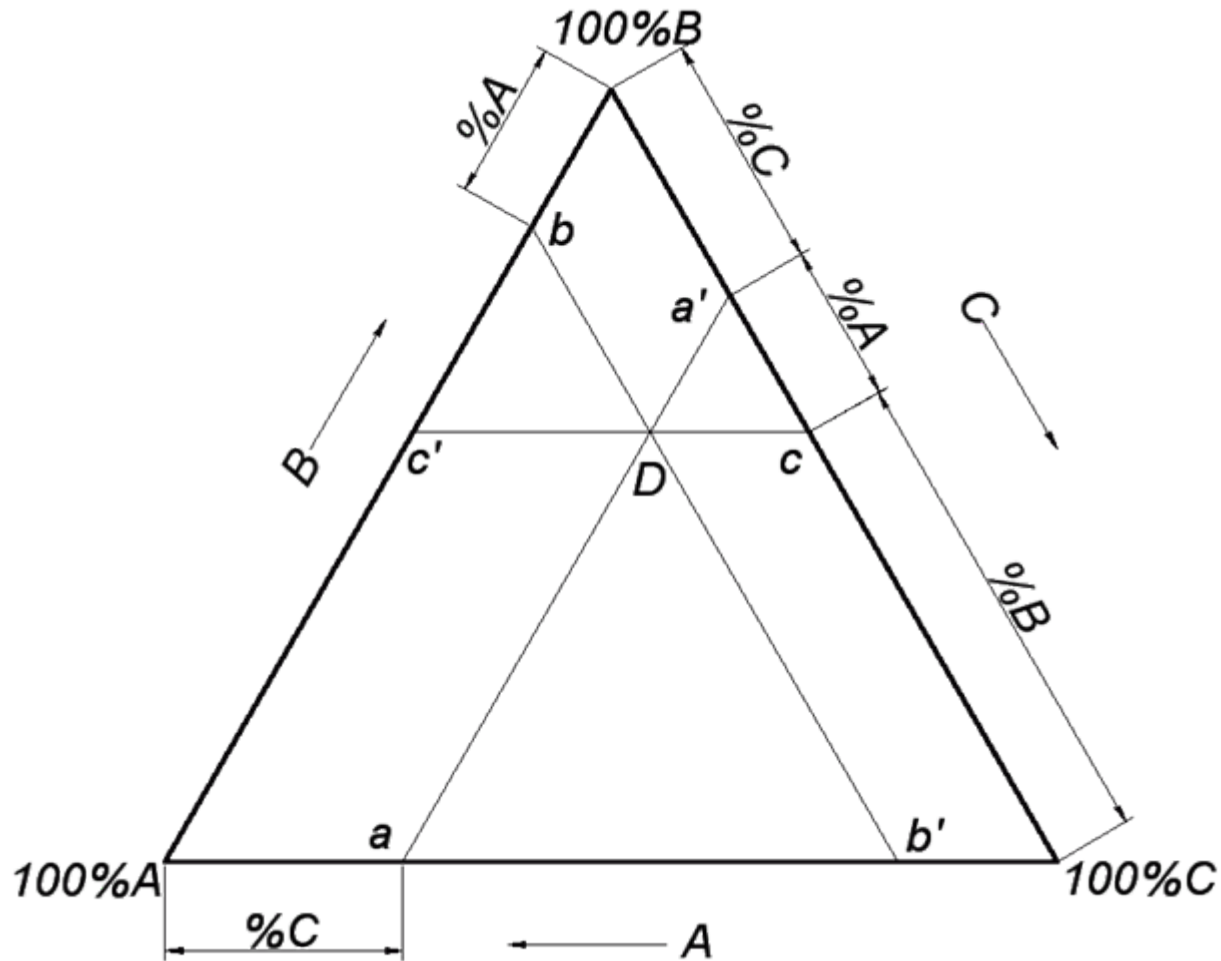
# Ternary systems



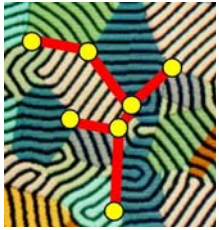
$$C\% = 100\% - A\% - B\%$$



# Ternary systems – composition triangle







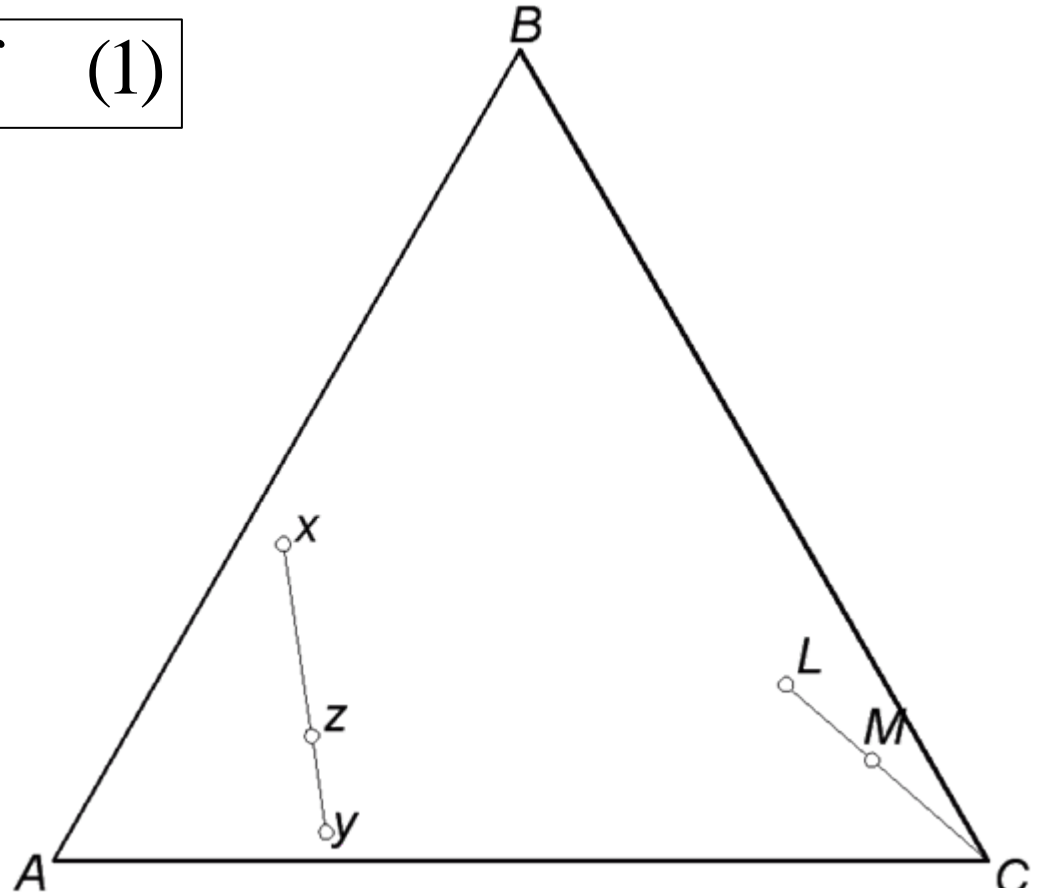
# Ternary systems – segment rule

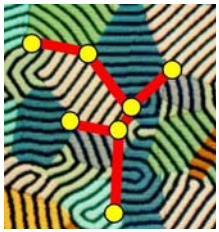


$$S = n - f + 1 = 4 - f \quad (1)$$

$$\frac{m_x}{m_y} = \frac{y - z}{x - z} \quad (2)$$

$$\frac{m_C}{m_L} = \frac{L - M}{M - C} \quad (3)$$

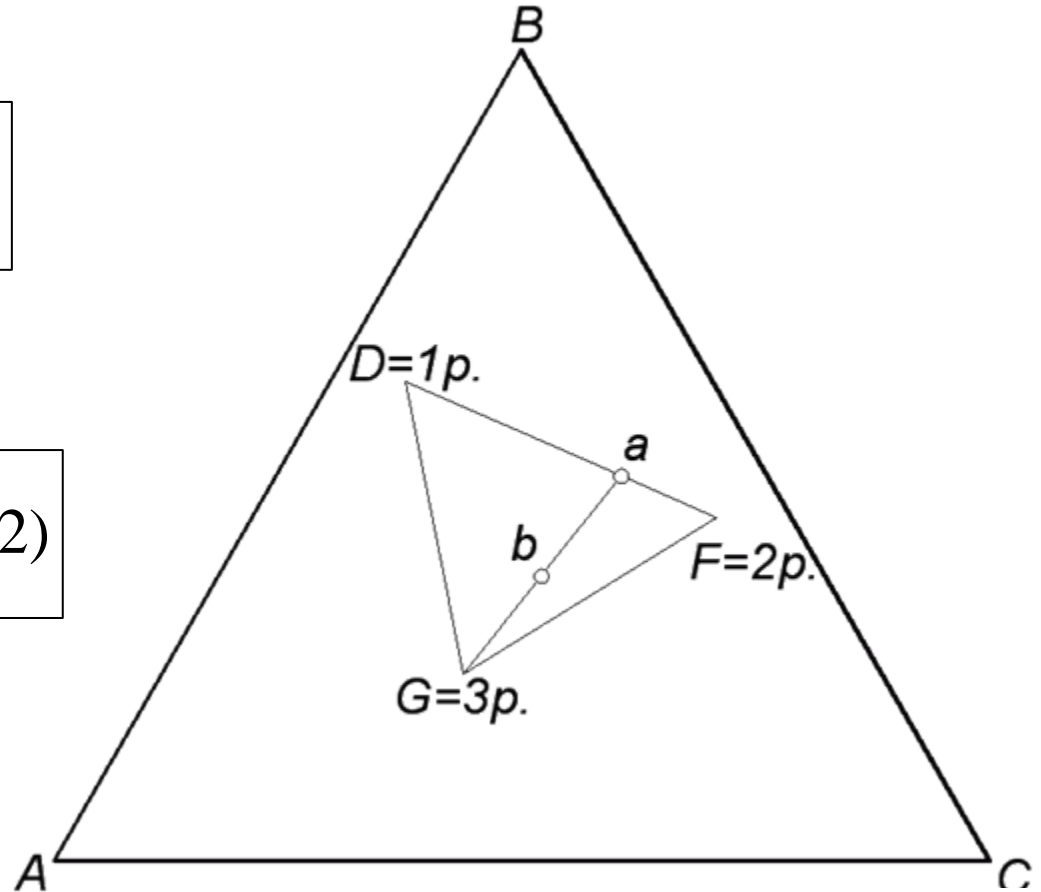


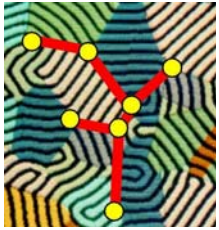


# Ternary systems

$$\frac{D}{F} = \frac{1}{2} = \frac{a-F}{a-D} \quad (1)$$

$$\frac{G}{D+F} = \frac{1}{1} = \frac{a-b}{b-G} \quad (2)$$





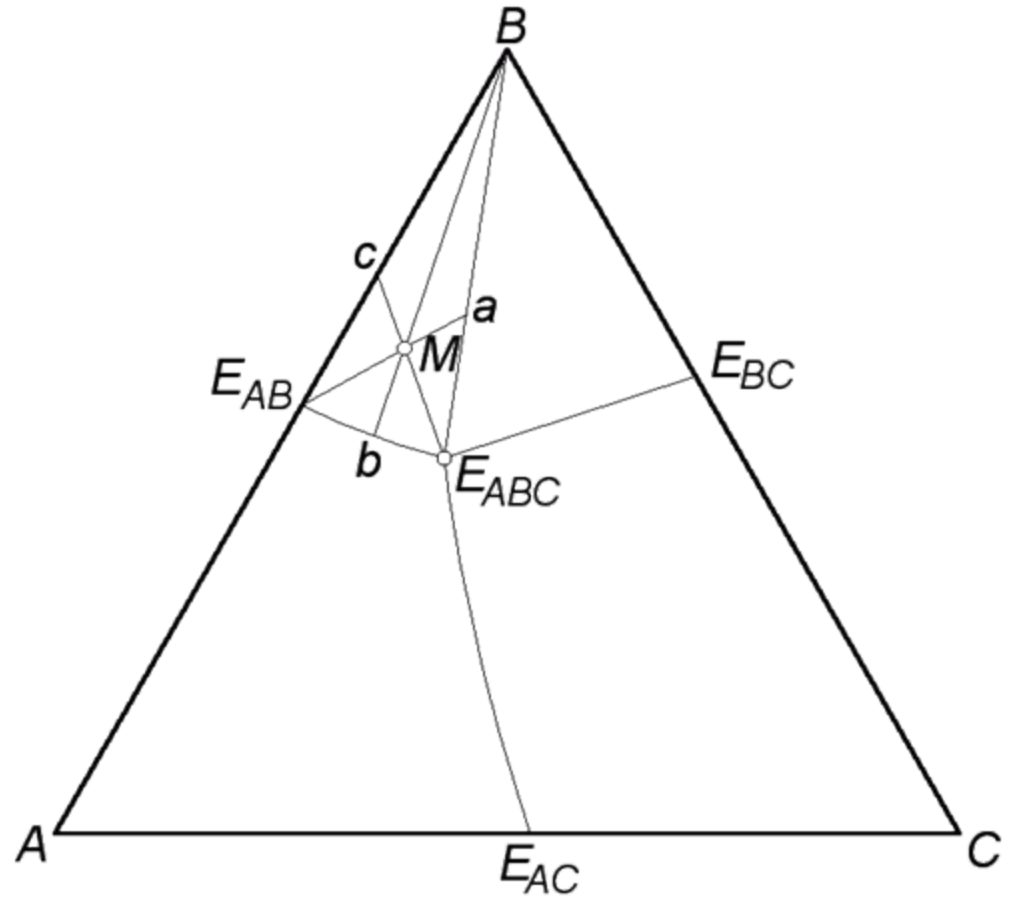
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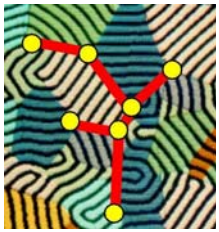


$$B = \frac{b - M}{B - b}$$

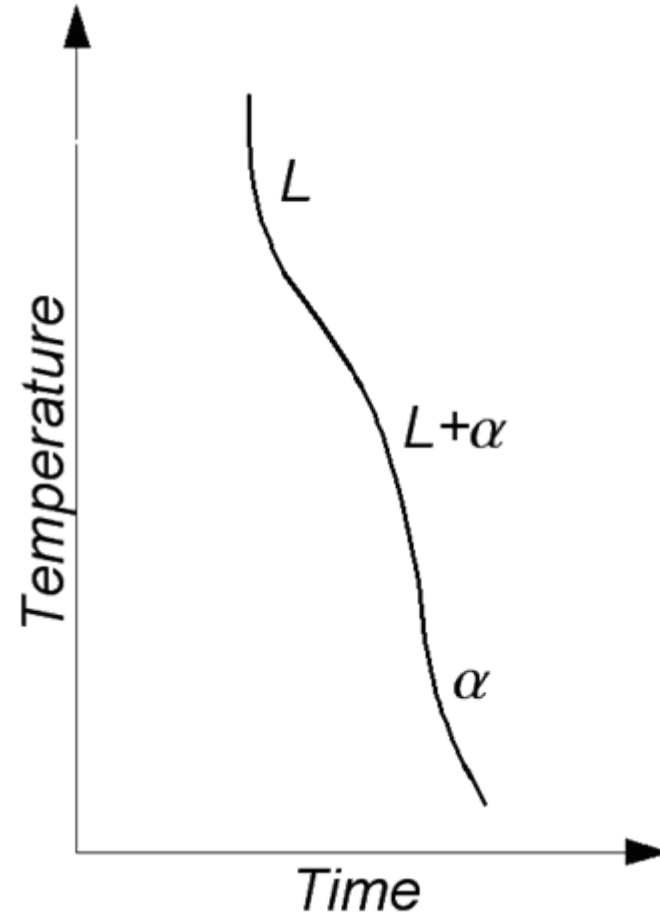
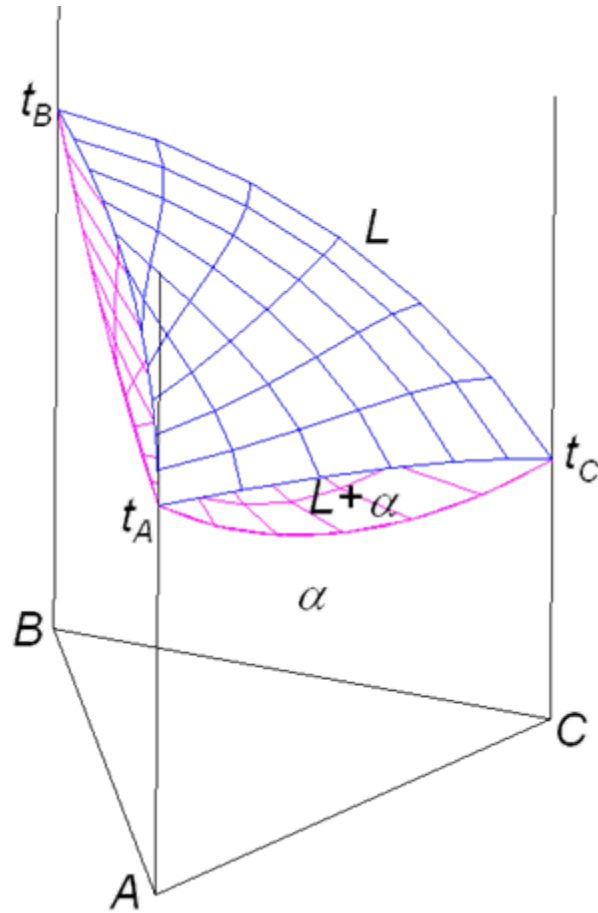
$$E_{AB} = \frac{a - M}{a - E_{AB}}$$

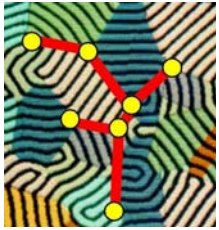
$$E_{ABC} = \frac{c - M}{c - E_{ABC}}$$



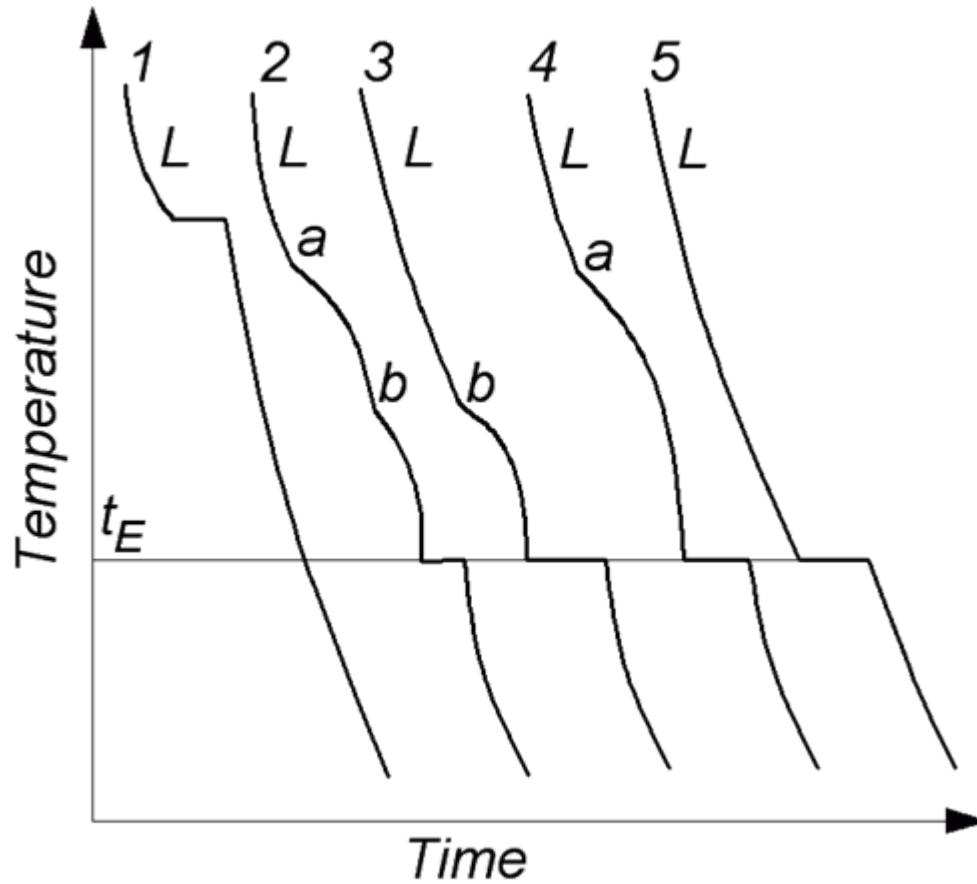


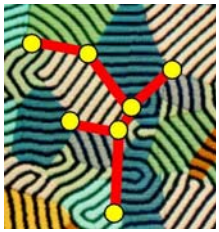
# Ternary systems – isomorphous system



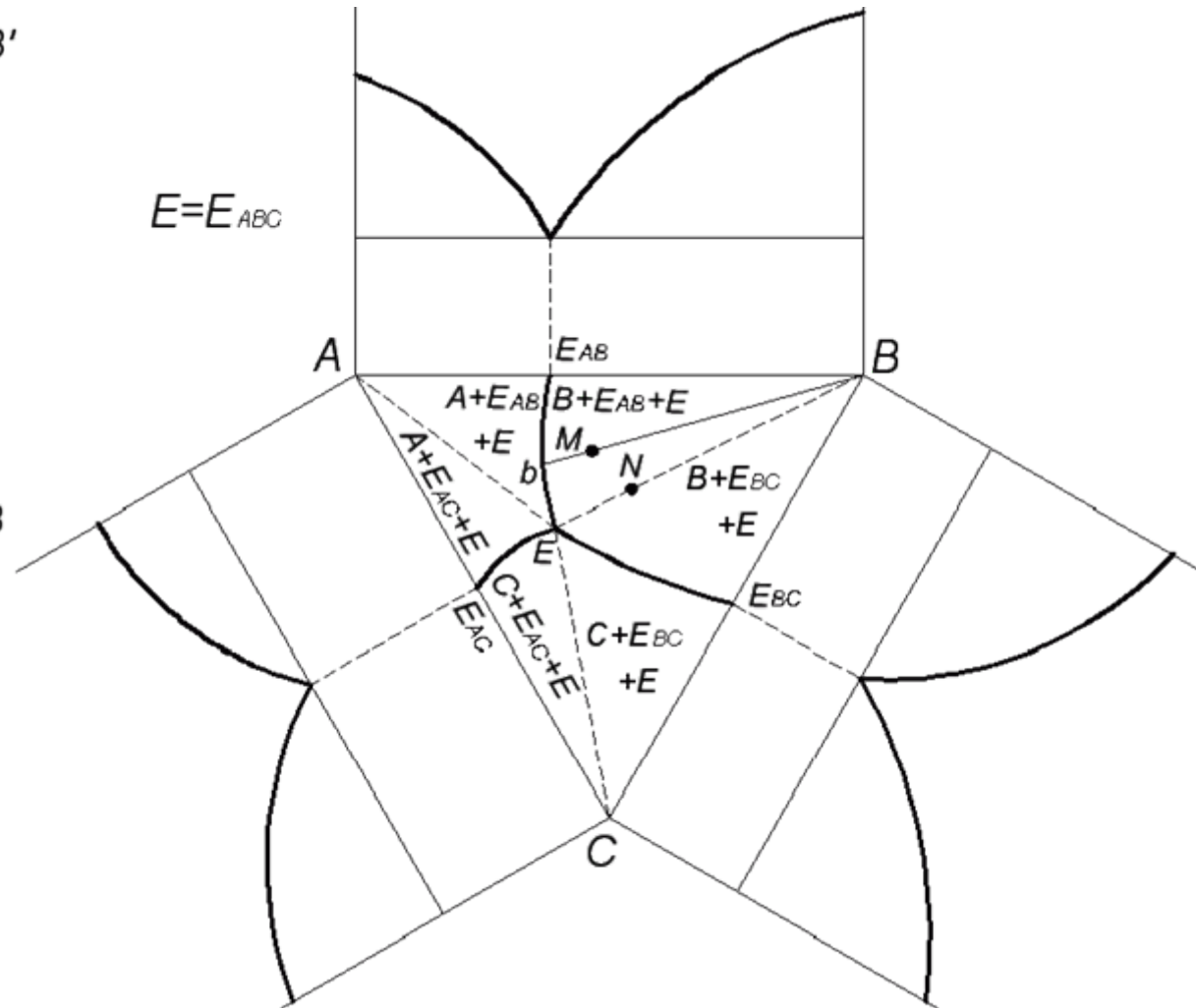
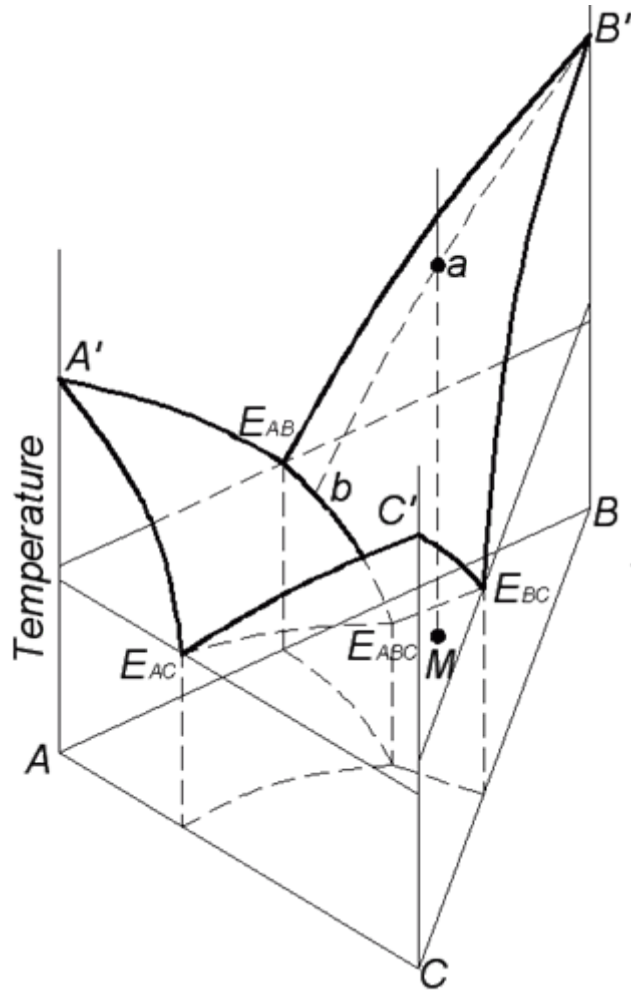


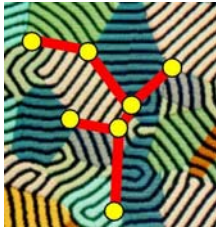
# Ternary systems – eutectic system





# Ternary systems – eutectic system

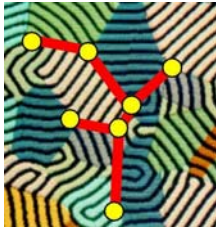




# Summary



- Basic concepts
- Gibbs free energy
- Lever rule
- Cooling curves
- Binary phase equilibrium diagrams
- Ternary phase equilibrium diagrams



# Literature



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- B. Chalmers, *Principles of Solidification*, Krieger Pub Co, 1977
- A.G. Guy, *Introduction to Materials Science*, McGraw-Hill, 1972
- W. Kurz, D.J. Fisher, *Fundamentals of solidification*, Trans Tech Publications, 1998